Maple 2018.2 Integration Test Results
on the problems in "7 Inverse hyperbolic functions/7.4 Inverse hyperbolic cotangent"

Test results for the 80 problems in "7.4.1 Inverse hyperbolic cotangent functions.txt"

Problem 6: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{arccoth}(a x)^2 \, dx$$

Optimal(type 3, 48 leaves, 5 steps):

$$\frac{x \operatorname{arccoth}(a x)}{a} - \frac{\operatorname{arccoth}(a x)^2}{2 a^2} + \frac{x^2 \operatorname{arccoth}(a x)^2}{2} + \frac{\ln(-a^2 x^2 + 1)}{2 a^2}$$

Result(type 3, 154 leaves):

$$\frac{x^2 \operatorname{arccoth}(a x)^2}{2} + \frac{x \operatorname{arccoth}(a x)}{a} + \frac{\operatorname{arccoth}(a x) \ln(a x - 1)}{2 a^2} - \frac{\operatorname{arccoth}(a x) \ln(a x + 1)}{2 a^2} + \frac{\ln(a x - 1)^2}{8 a^2} - \frac{\ln(a x - 1) \ln\left(\frac{a x}{2} + \frac{1}{2}\right)}{4 a^2} + \frac{\ln(a x - 1)}{2 a^2} + \frac{\ln(a x + 1)}{2 a^2} + \frac{\ln\left(-\frac{a x}{2} + \frac{1}{2}\right) \ln\left(\frac{a x}{2} + \frac{1}{2}\right)}{4 a^2} - \frac{\ln\left(-\frac{a x}{2} + \frac{1}{2}\right) \ln(a x + 1)}{4 a^2} + \frac{\ln(a x + 1)^2}{8 a^2}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \operatorname{arccoth}(a x)^2 \, dx$$

Optimal(type 4, 58 leaves, 5 steps):

$$\frac{\operatorname{arccoth}(a x)^2}{a} + x \operatorname{arccoth}(a x)^2 - \frac{2 \operatorname{arccoth}(a x) \ln\left(\frac{2}{-a x + 1}\right)}{a} - \frac{\operatorname{polylog}\left(2, 1 - \frac{2}{-a x + 1}\right)}{a}$$

Result(type 4, 121 leaves):

$$x \operatorname{arccoth}(a x)^2 - \frac{2 \operatorname{arccoth}(a x) \ln\left(1 - \frac{1}{\sqrt{\frac{a x - 1}{a x + 1}}}\right)}{a} - \frac{2 \operatorname{arccoth}(a x) \ln\left(1 + \frac{1}{\sqrt{\frac{a x - 1}{a x + 1}}}\right)}{a} + \frac{\operatorname{arccoth}(a x)^2}{a} - \frac{2 \operatorname{polylog}\left(2, \frac{1}{\sqrt{\frac{a x - 1}{a x + 1}}}\right)}{a} - \frac{2 \operatorname{polylog}\left(2, -\frac{1}{\sqrt{\frac{a x - 1}{a x + 1}}}\right)}{a}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{arccoth}(a x)^2}{x}\, dx$$

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Optimal(type 4, 93 leaves, 6 steps):
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$$2 \operatorname{arccoth}(a x)^2 \operatorname{arccoth}\left(1 - \frac{2}{-a x + 1}\right) + \operatorname{arccoth}(a x) \operatorname{polylog}\left(2, 1 - \frac{2}{a x + 1}\right) - \operatorname{arccoth}(a x) \operatorname{polylog}\left(2, 1 - \frac{2 a x}{a x + 1}\right) + \frac{\operatorname{polylog}\left(3, 1 - \frac{2}{a x + 1}\right)}{2} - \frac{\operatorname{polylog}\left(3, 1 - \frac{2 a x}{a x + 1}\right)}{2}$$

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Result(type 4, 486 leaves):
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$$\ln(a x) \operatorname{arccoth}(a x)^2 + \frac{I \pi \operatorname{csgn}\left(\frac{I}{\frac{a x + 1}{a x - 1} - 1}\right) \operatorname{csgn}\left(I \left(1 + \frac{a x + 1}{a x - 1}\right)\right) \operatorname{csgn}\left(\frac{I \left(1 + \frac{a x + 1}{a x - 1}\right)}{\frac{a x + 1}{a x - 1} - 1}\right) \operatorname{arccoth}(a x)^2}{2}$$
$$- \frac{I \pi \operatorname{csgn}\left(\frac{I}{\frac{a x + 1}{a x - 1} - 1}\right) \operatorname{csgn}\left(\frac{I \left(1 + \frac{a x + 1}{a x - 1}\right)}{\frac{a x + 1}{a x - 1} - 1}\right)^2 \operatorname{arccoth}(a x)^2}{2} - \frac{I \pi \operatorname{csgn}\left(I \left(1 + \frac{a x + 1}{a x - 1}\right)\right) \operatorname{csgn}\left(\frac{I \left(1 + \frac{a x + 1}{a x - 1}\right)}{\frac{a x + 1}{a x - 1} - 1}\right)^2 \operatorname{arccoth}(a x)^2}{2}$$
$$+ \frac{I \pi \operatorname{csgn}\left(\frac{I \left(1 + \frac{a x + 1}{a x - 1}\right)}{\frac{a x + 1}{a x - 1} - 1}\right)^3 \operatorname{arccoth}(a x)^2}{2} + \operatorname{arccoth}(a x)^2 \ln\left(\frac{a x + 1}{a x - 1} - 1\right) - \operatorname{arccoth}(a x)^2 \ln\left(1 + \frac{1}{\sqrt{\frac{a x - 1}{a x + 1}}}\right) - 2 \operatorname{arccoth}(a x) \operatorname{polylog}\left(2, -\frac{1}{\sqrt{\frac{a x - 1}{a x + 1}}}\right) + 2 \operatorname{polylog}\left(3, -\frac{1}{\sqrt{\frac{a x - 1}{a x + 1}}}\right) - \operatorname{arccoth}(a x)^2 \ln\left(1 - \frac{1}{\sqrt{\frac{a x - 1}{a x + 1}}}\right) - 2 \operatorname{arccoth}(a x) \operatorname{polylog}\left(2, \frac{1}{\sqrt{\frac{a x - 1}{a x + 1}}}\right) + 2 \operatorname{polylog}\left(3, \frac{1}{\sqrt{\frac{a x - 1}{a x + 1}}}\right) + \operatorname{arccoth}(a x) \operatorname{polylog}\left(2, -\frac{a x + 1}{a x - 1}\right) - \frac{\operatorname{polylog}\left(3, -\frac{a x + 1}{a x - 1}\right)}{2}$$

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Problem 9: Result more than twice size of optimal antiderivative.
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$$\int \frac{\operatorname{arccoth}(a x)^2}{x^2}\, dx$$

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Optimal(type 4, 55 leaves, 4 steps):
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$$a\,\text{arccoth}(a x)^2-\frac{\text{arccoth}(a x)^2}{x}+2 a\,\text{arccoth}(a x) \ln\left(2-\frac{2}{a x+1}\right)-a\,\text{polylog}\left(2, -1+\frac{2}{a x+1}\right)$$

Result(type 4, 158 leaves):

$$-\frac{\text{arccoth}(a x)^2}{x}-a\,\text{arccoth}(a x) \ln(a x+1)-a\,\text{arccoth}(a x) \ln(a x-1)+2 a \ln(a x) \text{arccoth}(a x)-\frac{a \ln(a x-1)^2}{4}+a\,\text{dilog}\left(\frac{a x}{2}+\frac{1}{2}\right)+\frac{a \ln(a x-1) \ln\left(\frac{a x}{2}+\frac{1}{2}\right)}{2}-\frac{a \ln\left(-\frac{a x}{2}+\frac{1}{2}\right) \ln(a x+1)}{2}+\frac{a \ln\left(-\frac{a x}{2}+\frac{1}{2}\right) \ln\left(\frac{a x}{2}+\frac{1}{2}\right)}{2}+\frac{a \ln(a x+1)^2}{4}-a\,\text{dilog}(a x+1)-a \ln(a x) \ln(a x+1)-a\,\text{dilog}(a x)$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int x^5\,\text{arccoth}(a x)^3\,dx$$

Optimal(type 4, 160 leaves, 33 steps):

$$\frac{19 x}{60 a^5}+\frac{x^3}{60 a^3}+\frac{4 x^2 \text{arccoth}(a x)}{15 a^4}+\frac{x^4 \text{arccoth}(a x)}{20 a^2}+\frac{23\,\text{arccoth}(a x)^2}{30 a^6}+\frac{x\,\text{arccoth}(a x)^2}{2 a^5}+\frac{x^3 \text{arccoth}(a x)^2}{6 a^3}+\frac{x^5 \text{arccoth}(a x)^2}{10 a}-\frac{\text{arccoth}(a x)^3}{6 a^6}+\frac{x^6 \text{arccoth}(a x)^3}{6}-\frac{19\,\text{arctanh}(a x)}{60 a^6}-\frac{23\,\text{arccoth}(a x) \ln\left(\frac{2}{-a x+1}\right)}{15 a^6}-\frac{23\,\text{polylog}\left(2, 1-\frac{2}{-a x+1}\right)}{30 a^6}$$

Result(type 4, 1140 leaves):

$$\frac{23\,\text{arccoth}(a x)^2}{30 a^6}-\frac{\text{arccoth}(a x)^3}{6 a^6}+\frac{x^6 \text{arccoth}(a x)^3}{6}+\frac{4 x^2 \text{arccoth}(a x)}{15 a^4}+\frac{x^4 \text{arccoth}(a x)}{20 a^2}+\frac{x\,\text{arccoth}(a x)^2}{2 a^5}+\frac{x^3 \text{arccoth}(a x)^2}{6 a^3}+\frac{x^5 \text{arccoth}(a x)^2}{10 a}-\frac{I \pi\,\text{csgn}\left(\frac{I (a x+1)}{a x-1}\right) \text{csgn}\left(\frac{I (a x+1)}{(a x-1)\left(\frac{a x+1}{a x-1}-1\right)}\right) \text{csgn}\left(\frac{I}{\frac{a x+1}{a x-1}-1}\right) \text{arccoth}(a x)^2}{8 a^6}+\frac{x}{80 a^5\left(\sqrt{\frac{a x-1}{a x+1}}\,a x+\sqrt{\frac{a x-1}{a x+1}}+a x\right)}-\frac{23\,\text{arccoth}(a x) \ln\left(1+\frac{1}{\sqrt{\frac{a x-1}{a x+1}}}\right)}{15 a^6}-\frac{41 \sqrt{\frac{a x-1}{a x+1}}}{120 a^6\left(\sqrt{\frac{a x-1}{a x+1}}-1\right)}-\frac{41 \sqrt{\frac{a x-1}{a x+1}}}{120 a^6\left(\sqrt{\frac{a x-1}{a x+1}}+1\right)}+\frac{\text{arccoth}(a x)^2 \ln(a x-1)}{4 a^6}-\frac{\text{arccoth}(a x)^2 \ln(a x+1)}{4 a^6}-\frac{\text{arccoth}(a x)^2 \ln\left(\frac{a x-1}{a x+1}\right)}{4 a^6}+\frac{\sqrt{\frac{a x-1}{a x+1}}}{120 a^6\left(2 \sqrt{\frac{a x-1}{a x+1}}\,a x+\sqrt{\frac{a x-1}{a x+1}}-2 a x+1\right)}$$

$$+\frac{\sqrt{\frac{ax-1}{ax+1}}}{120a^6\left(2\sqrt{\frac{ax-1}{ax+1}}\,ax+\sqrt{\frac{ax-1}{ax+1}}+2ax-1\right)}-\frac{x}{80a^5\left(\sqrt{\frac{ax-1}{ax+1}}\,ax+\sqrt{\frac{ax-1}{ax+1}}-ax\right)}+\frac{1}{80a^6\left(\sqrt{\frac{ax-1}{ax+1}}\,ax+\sqrt{\frac{ax-1}{ax+1}}-ax\right)}$$

$$-\frac{1}{80a^6\left(\sqrt{\frac{ax-1}{ax+1}}\,ax+\sqrt{\frac{ax-1}{ax+1}}+ax\right)}-\frac{23\,\mathrm{dilog}\left(1+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)}{15a^6}+\frac{23\,\mathrm{dilog}\left(\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)}{15a^6}-\frac{19\,\mathrm{arccoth}(ax)}{60a^6}$$

$$-\frac{I\pi\,\mathrm{csgn}\left(\frac{I(ax+1)}{ax-1}\right)^3\mathrm{arccoth}(ax)^2}{8a^6}-\frac{I\pi\,\mathrm{csgn}\left(\frac{I(ax+1)}{(ax-1)\left(\frac{ax+1}{ax-1}-1\right)}\right)^3\mathrm{arccoth}(ax)^2}{8a^6}$$

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,x}{120a^5\left(2\sqrt{\frac{ax-1}{ax+1}}\,ax+\sqrt{\frac{ax-1}{ax+1}}-2ax+1\right)}-\frac{\sqrt{\frac{ax-1}{ax+1}}\,x}{120a^5\left(2\sqrt{\frac{ax-1}{ax+1}}\,ax+\sqrt{\frac{ax-1}{ax+1}}+2ax-1\right)}$$

$$+\frac{I\pi\,\mathrm{csgn}\left(\frac{I}{\sqrt{\frac{ax-1}{ax+1}}}\right)\mathrm{csgn}\left(\frac{I(ax+1)}{ax-1}\right)^2\mathrm{arccoth}(ax)^2}{4a^6}+\frac{I\pi\,\mathrm{csgn}\left(\frac{I(ax+1)}{ax-1}\right)\mathrm{csgn}\left(\frac{I(ax+1)}{(ax-1)\left(\frac{ax+1}{ax-1}-1\right)}\right)^2\mathrm{arccoth}(ax)^2}{8a^6}$$

$$+\frac{I\pi\,\mathrm{csgn}\left(\frac{I(ax+1)}{(ax-1)\left(\frac{ax+1}{ax-1}-1\right)}\right)^2\mathrm{csgn}\left(\frac{I}{\frac{ax+1}{ax-1}-1}\right)\mathrm{arccoth}(ax)^2}{8a^6}-\frac{I\pi\,\mathrm{csgn}\left(\frac{I}{\sqrt{\frac{ax-1}{ax+1}}}\right)^2\mathrm{csgn}\left(\frac{I(ax+1)}{ax-1}\right)\mathrm{arccoth}(ax)^2}{8a^6}$$

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Problem 11: Result more than twice size of optimal antiderivative.
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$$\int\frac{\mathrm{arccoth}(ax)^3}{x}\,dx$$

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Optimal(type 4, 138 leaves, 8 steps):
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$$2\,\mathrm{arccoth}(ax)^3\,\mathrm{arccoth}\left(1-\frac{2}{-ax+1}\right)+\frac{3\,\mathrm{arccoth}(ax)^2\,\mathrm{polylog}\left(2,1-\frac{2}{ax+1}\right)}{2}-\frac{3\,\mathrm{arccoth}(ax)^2\,\mathrm{polylog}\left(2,1-\frac{2ax}{ax+1}\right)}{2}$$

$$+\frac{3\,\mathrm{arccoth}(ax)\,\mathrm{polylog}\left(3,1-\frac{2}{ax+1}\right)}{2}-\frac{3\,\mathrm{arccoth}(ax)\,\mathrm{polylog}\left(3,1-\frac{2ax}{ax+1}\right)}{2}+\frac{3\,\mathrm{polylog}\left(4,1-\frac{2}{ax+1}\right)}{4}-\frac{3\,\mathrm{polylog}\left(4,1-\frac{2ax}{ax+1}\right)}{4}$$

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Result(type 4, 563 leaves):
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$$\ln(a x) \operatorname{arccoth}(a x)^3 + \operatorname{arccoth}(a x)^3 \ln\left(\frac{a x+1}{a x-1}-1\right) + \frac{3 \operatorname{arccoth}(a x)^2 \operatorname{polylog}\left(2, -\frac{a x+1}{a x-1}\right)}{2} - \frac{3 \operatorname{arccoth}(a x) \operatorname{polylog}\left(3, -\frac{a x+1}{a x-1}\right)}{2}$$

$$+ \frac{3 \operatorname{polylog}\left(4, -\frac{a x+1}{a x-1}\right)}{4} - \frac{I \pi \operatorname{csgn}\left(I\left(1+\frac{a x+1}{a x-1}\right)\right) \operatorname{csgn}\left(\frac{I\left(1+\frac{a x+1}{a x-1}\right)}{\frac{a x+1}{a x-1}-1}\right)^2 \operatorname{arccoth}(a x)^3}{2}$$

$$- \frac{I \pi \operatorname{csgn}\left(\frac{I}{\frac{a x+1}{a x-1}-1}\right) \operatorname{csgn}\left(\frac{I\left(1+\frac{a x+1}{a x-1}\right)}{\frac{a x+1}{a x-1}-1}\right)^2 \operatorname{arccoth}(a x)^3}{2} + \frac{I \pi \operatorname{csgn}\left(\frac{I\left(1+\frac{a x+1}{a x-1}\right)}{\frac{a x+1}{a x-1}-1}\right)^3 \operatorname{arccoth}(a x)^3}{2}$$

$$+ \frac{I \pi \operatorname{csgn}\left(\frac{I}{\frac{a x+1}{a x-1}-1}\right) \operatorname{csgn}\left(I\left(1+\frac{a x+1}{a x-1}\right)\right) \operatorname{csgn}\left(\frac{I\left(1+\frac{a x+1}{a x-1}\right)}{\frac{a x+1}{a x-1}-1}\right) \operatorname{arccoth}(a x)^3}{2} - \operatorname{arccoth}(a x)^3 \ln\left(1+\frac{1}{\sqrt{\frac{a x-1}{a x+1}}}\right)$$

$$- 3 \operatorname{arccoth}(a x)^2 \operatorname{polylog}\left(2, -\frac{1}{\sqrt{\frac{a x-1}{a x+1}}}\right) + 6 \operatorname{arccoth}(a x) \operatorname{polylog}\left(3, -\frac{1}{\sqrt{\frac{a x-1}{a x+1}}}\right) - 6 \operatorname{polylog}\left(4, -\frac{1}{\sqrt{\frac{a x-1}{a x+1}}}\right) - \operatorname{arccoth}(a x)^3 \ln\left(1\right.$$

$$\left. - \frac{1}{\sqrt{\frac{a x-1}{a x+1}}}\right) - 3 \operatorname{arccoth}(a x)^2 \operatorname{polylog}\left(2, \frac{1}{\sqrt{\frac{a x-1}{a x+1}}}\right) + 6 \operatorname{arccoth}(a x) \operatorname{polylog}\left(3, \frac{1}{\sqrt{\frac{a x-1}{a x+1}}}\right) - 6 \operatorname{polylog}\left(4, \frac{1}{\sqrt{\frac{a x-1}{a x+1}}}\right)$$

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Problem 13: Result is not expressed in closed-form.
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$$\int \frac{\operatorname{arccoth}(a x)}{d x^2 + c}\, dx$$

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Optimal(type 4, 280 leaves, 27 steps):
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$$-\frac{\arctan\left(\frac{x\sqrt{d}}{\sqrt{c}}\right) \ln\left(1-\frac{1}{a x}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\arctan\left(\frac{x\sqrt{d}}{\sqrt{c}}\right) \ln\left(1+\frac{1}{a x}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\arctan\left(\frac{x\sqrt{d}}{\sqrt{c}}\right) \ln\left(-\frac{2(-a x+1)\sqrt{c}\sqrt{d}}{\left(I a\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-I x\sqrt{d}\right)}\right)}{2\sqrt{c}\sqrt{d}}$$

$$- \frac{\arctan\left(\frac{x\sqrt{d}}{\sqrt{c}}\right) \ln\left(\frac{2(a x+1)\sqrt{c}\sqrt{d}}{\left(I a\sqrt{c}+\sqrt{d}\right)\left(\sqrt{c}-I x\sqrt{d}\right)}\right)}{2\sqrt{c}\sqrt{d}} - \frac{I \operatorname{polylog}\left(2, 1+\frac{2(-a x+1)\sqrt{c}\sqrt{d}}{\left(I a\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-I x\sqrt{d}\right)}\right)}{4\sqrt{c}\sqrt{d}}$$

$$+\frac{I\,\mathrm{polylog}\left(2, 1-\frac{2\,(a x+1)\sqrt{c}\,\sqrt{d}}{\left(I a\sqrt{c}+\sqrt{d}\right)\left(\sqrt{c}-I x\sqrt{d}\right)}\right)}{4\sqrt{c}\,\sqrt{d}}$$

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Result(type 7, 117 leaves):
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$$-a\left(\sum_{\_R1=RootOf\left((c a^2+d)\,\_Z^4+(-2 c a^2+2 d)\,\_Z^2+c a^2+d\right)}\frac{\mathrm{arccoth}(a x)\ln\left(\frac{\_R1-\frac{1}{\sqrt{\frac{a x-1}{a x+1}}}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-\frac{1}{\sqrt{\frac{a x-1}{a x+1}}}}{\_R1}\right)}{\_R1^2 a^2 c+\_R1^2 d-c a^2+d}\right)$$

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Problem 14: Result is not expressed in closed-form.
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$$\int\frac{\mathrm{arccoth}(a x)}{(d x^2+c)^3}\,dx$$

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Optimal(type 4, 493 leaves, 23 steps):
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$$\frac{a}{8 c\,(c a^2+d)\,(d x^2+c)}+\frac{x\,\mathrm{arccoth}(a x)}{4 c\,(d x^2+c)^2}+\frac{3 x\,\mathrm{arccoth}(a x)}{8 c^2\,(d x^2+c)}+\frac{a\,(5 c a^2+3 d)\ln(-a^2 x^2+1)}{16 c^2\,(c a^2+d)^2}-\frac{a\,(5 c a^2+3 d)\ln(d x^2+c)}{16 c^2\,(c a^2+d)^2}$$
$$+\frac{3\,\mathrm{arccoth}(a x)\arctan\left(\frac{x\sqrt{d}}{\sqrt{c}}\right)}{8 c^{5/2}\sqrt{d}}-\frac{3 I\ln\left(-\frac{(a x+1)\sqrt{d}}{I a\sqrt{c}-\sqrt{d}}\right)\ln\left(1-\frac{I x\sqrt{d}}{\sqrt{c}}\right)}{32 c^{5/2}\sqrt{d}}+\frac{3 I\ln\left(\frac{(-a x+1)\sqrt{d}}{I a\sqrt{c}+\sqrt{d}}\right)\ln\left(1-\frac{I x\sqrt{d}}{\sqrt{c}}\right)}{32 c^{5/2}\sqrt{d}}$$
$$-\frac{3 I\ln\left(-\frac{(-a x+1)\sqrt{d}}{I a\sqrt{c}-\sqrt{d}}\right)\ln\left(1+\frac{I x\sqrt{d}}{\sqrt{c}}\right)}{32 c^{5/2}\sqrt{d}}+\frac{3 I\ln\left(\frac{(a x+1)\sqrt{d}}{I a\sqrt{c}+\sqrt{d}}\right)\ln\left(1+\frac{I x\sqrt{d}}{\sqrt{c}}\right)}{32 c^{5/2}\sqrt{d}}+\frac{3 I\,\mathrm{polylog}\left(2, \frac{a\left(\sqrt{c}-I x\sqrt{d}\right)}{a\sqrt{c}-I\sqrt{d}}\right)}{32 c^{5/2}\sqrt{d}}$$
$$-\frac{3 I\,\mathrm{polylog}\left(2, \frac{a\left(\sqrt{c}-I x\sqrt{d}\right)}{a\sqrt{c}+I\sqrt{d}}\right)}{32 c^{5/2}\sqrt{d}}+\frac{3 I\,\mathrm{polylog}\left(2, \frac{a\left(\sqrt{c}+I x\sqrt{d}\right)}{a\sqrt{c}-I\sqrt{d}}\right)}{32 c^{5/2}\sqrt{d}}-\frac{3 I\,\mathrm{polylog}\left(2, \frac{a\left(\sqrt{c}+I x\sqrt{d}\right)}{a\sqrt{c}+I\sqrt{d}}\right)}{32 c^{5/2}\sqrt{d}}$$

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Result(type ?, 2849 leaves): Display of huge result suppressed!
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Problem 15: Unable to integrate problem.
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$$\int\frac{\mathrm{arccoth}(a x)}{(d x^2+c)^{5/2}}\,dx$$

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Optimal(type 3, 108 leaves, 7 steps):
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$$\frac{x\,\mathrm{arccoth}(a x)}{3 c \left(d x^2+c\right)^{3/2}}-\frac{\left(3 c a^2+2 d\right) \mathrm{arctanh}\left(\frac{a \sqrt{d x^2+c}}{\sqrt{c a^2+d}}\right)}{3 c^2 \left(c a^2+d\right)^{3/2}}+\frac{a}{3 c \left(c a^2+d\right) \sqrt{d x^2+c}}+\frac{2 x\,\mathrm{arccoth}(a x)}{3 c^2 \sqrt{d x^2+c}}$$

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Result(type 8, 16 leaves):
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$$\int \frac{\mathrm{arccoth}(a x)}{\left(d x^2+c\right)^{5/2}}\, dx$$

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Problem 16: Unable to integrate problem.
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$$\int \frac{\mathrm{arccoth}(a x)}{\left(d x^2+c\right)^{9/2}}\, dx$$

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Optimal(type 3, 247 leaves, 8 steps):
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$$\frac{a}{35 c \left(c a^2+d\right) \left(d x^2+c\right)^{5/2}}+\frac{a \left(11 c a^2+6 d\right)}{105 c^2 \left(c a^2+d\right)^2 \left(d x^2+c\right)^{3/2}}+\frac{x\,\mathrm{arccoth}(a x)}{7 c \left(d x^2+c\right)^{7/2}}+\frac{6 x\,\mathrm{arccoth}(a x)}{35 c^2 \left(d x^2+c\right)^{5/2}}+\frac{8 x\,\mathrm{arccoth}(a x)}{35 c^3 \left(d x^2+c\right)^{3/2}}$$
$$-\frac{\left(35 c^3 a^6+70 a^4 c^2 d+56 a^2 c d^2+16 d^3\right) \mathrm{arctanh}\left(\frac{a \sqrt{d x^2+c}}{\sqrt{c a^2+d}}\right)}{35 c^4 \left(c a^2+d\right)^{7/2}}+\frac{a \left(19 c^2 a^4+22 c a^2 d+8 d^2\right)}{35 c^3 \left(c a^2+d\right)^3 \sqrt{d x^2+c}}+\frac{16 x\,\mathrm{arccoth}(a x)}{35 c^4 \sqrt{d x^2+c}}$$

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Result(type 8, 16 leaves):
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$$\int \frac{\mathrm{arccoth}(a x)}{\left(d x^2+c\right)^{9/2}}\, dx$$

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Problem 18: Result more than twice size of optimal antiderivative.
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$$\int \frac{x\,\mathrm{arccoth}(x)}{-x^2+1}\, dx$$

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Optimal(type 4, 33 leaves, 4 steps):
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$$-\frac{\mathrm{arccoth}(x)^2}{2}+\mathrm{arccoth}(x) \ln\left(\frac{2}{1-x}\right)+\frac{\mathrm{polylog}\left(2, \frac{1+x}{-1+x}\right)}{2}$$

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Result(type 4, 74 leaves):
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$$-\frac{\mathrm{arccoth}(x) \ln(-1+x)}{2}-\frac{\mathrm{arccoth}(x) \ln(1+x)}{2}-\frac{\ln(-1+x)^2}{8}+\frac{\mathrm{dilog}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}+\frac{\ln(-1+x) \ln\left(\frac{x}{2}+\frac{1}{2}\right)}{4}$$
$$-\frac{\left(\ln(1+x)-\ln\left(\frac{x}{2}+\frac{1}{2}\right)\right) \ln\left(-\frac{x}{2}+\frac{1}{2}\right)}{4}+\frac{\ln(1+x)^2}{8}$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{arccoth}(x)}{(-x^2+1)^2}\, dx$$

Optimal(type 3, 32 leaves, 2 steps):

$$-\frac{1}{4\,(-x^2+1)}+\frac{x\operatorname{arccoth}(x)}{2\,(-x^2+1)}+\frac{\operatorname{arccoth}(x)^2}{4}$$

Result(type 3, 98 leaves):

$$-\frac{\operatorname{arccoth}(x)}{4\,(1+x)}+\frac{\operatorname{arccoth}(x)\ln(1+x)}{4}-\frac{\operatorname{arccoth}(x)}{4\,(-1+x)}-\frac{\operatorname{arccoth}(x)\ln(-1+x)}{4}-\frac{\ln(-1+x)^2}{16}+\frac{1}{8\,(-1+x)}+\frac{\ln(-1+x)\ln\left(\frac{x}{2}+\frac{1}{2}\right)}{8}$$
$$+\frac{\left(\ln(1+x)-\ln\left(\frac{x}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{x}{2}+\frac{1}{2}\right)}{8}-\frac{\ln(1+x)^2}{16}-\frac{1}{8\,(1+x)}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int x^2\operatorname{arccoth}(b\,x+a)^2\, dx$$

Optimal(type 4, 190 leaves, 15 steps):

$$\frac{x}{3\,b^2}-\frac{2\,a\,(b\,x+a)\operatorname{arccoth}(b\,x+a)}{b^3}+\frac{(b\,x+a)^2\operatorname{arccoth}(b\,x+a)}{3\,b^3}+\frac{a\,(a^2+3)\operatorname{arccoth}(b\,x+a)^2}{3\,b^3}+\frac{(3\,a^2+1)\operatorname{arccoth}(b\,x+a)^2}{3\,b^3}$$
$$+\frac{x^3\operatorname{arccoth}(b\,x+a)^2}{3}-\frac{\operatorname{arctanh}(b\,x+a)}{3\,b^3}-\frac{2\,(3\,a^2+1)\operatorname{arccoth}(b\,x+a)\ln\left(\frac{2}{-b\,x-a+1}\right)}{3\,b^3}-\frac{a\ln\left(1-(b\,x+a)^2\right)}{b^3}$$
$$-\frac{(3\,a^2+1)\operatorname{polylog}\left(2,\frac{-b\,x-a-1}{-b\,x-a+1}\right)}{3\,b^3}$$

Result(type 4, 728 leaves):

$$\frac{x}{3\,b^2}+\frac{x^3\operatorname{arccoth}(b\,x+a)^2}{3}+\frac{\operatorname{arccoth}(b\,x+a)\,x^2}{3\,b}-\frac{\ln(b\,x+a-1)\ln\left(\frac{b\,x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{6\,b^3}+\frac{\operatorname{arccoth}(b\,x+a)\ln(b\,x+a-1)}{3\,b^3}$$
$$+\frac{\operatorname{arccoth}(b\,x+a)\ln(b\,x+a+1)}{3\,b^3}+\frac{\ln\left(-\frac{b\,x}{2}-\frac{a}{2}+\frac{1}{2}\right)\ln(b\,x+a+1)}{6\,b^3}-\frac{\ln\left(-\frac{b\,x}{2}-\frac{a}{2}+\frac{1}{2}\right)\ln\left(\frac{b\,x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{6\,b^3}-\frac{a^2\ln(b\,x+a+1)^2}{4\,b^3}$$
$$-\frac{a^3\ln(b\,x+a-1)^2}{12\,b^3}-\frac{a\ln(b\,x+a+1)^2}{4\,b^3}-\frac{a\ln(b\,x+a-1)^2}{4\,b^3}+\frac{a^2\ln(b\,x+a-1)^2}{4\,b^3}-\frac{a^2\operatorname{dilog}\left(\frac{b\,x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{b^3}-\frac{a^3\ln(b\,x+a+1)^2}{12\,b^3}$$

$$-\frac{\ln(bx+a-1)\,a}{b^3}-\frac{\ln(bx+a+1)\,a}{b^3}-\frac{5\,\text{arccoth}(bx+a)\,a^2}{3\,b^3}+\frac{a}{3\,b^3}-\frac{\ln(bx+a+1)^2}{12\,b^3}+\frac{\ln(bx+a-1)^2}{12\,b^3}-\frac{\text{dilog}\left(\frac{bx}{2}+\frac{a}{2}+\frac{1}{2}\right)}{3\,b^3}$$
$$+\frac{\ln(bx+a-1)}{6\,b^3}-\frac{\ln(bx+a+1)}{6\,b^3}-\frac{\text{arccoth}(bx+a)\ln(bx+a-1)\,a^3}{3\,b^3}+\frac{\text{arccoth}(bx+a)\ln(bx+a-1)\,a^2}{b^3}$$
$$-\frac{\text{arccoth}(bx+a)\ln(bx+a-1)\,a}{b^3}+\frac{\text{arccoth}(bx+a)\ln(bx+a+1)\,a^3}{3\,b^3}+\frac{\text{arccoth}(bx+a)\ln(bx+a+1)\,a^2}{b^3}$$
$$+\frac{\text{arccoth}(bx+a)\ln(bx+a+1)\,a}{b^3}-\frac{a^3\ln\left(-\frac{bx}{2}-\frac{a}{2}+\frac{1}{2}\right)\ln\left(\frac{bx}{2}+\frac{a}{2}+\frac{1}{2}\right)}{6\,b^3}+\frac{a^3\ln\left(-\frac{bx}{2}-\frac{a}{2}+\frac{1}{2}\right)\ln(bx+a+1)}{6\,b^3}$$
$$+\frac{a\ln(bx+a-1)\ln\left(\frac{bx}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2\,b^3}-\frac{a^2\ln(bx+a-1)\ln\left(\frac{bx}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2\,b^3}+\frac{a^3\ln(bx+a-1)\ln\left(\frac{bx}{2}+\frac{a}{2}+\frac{1}{2}\right)}{6\,b^3}$$
$$-\frac{a\ln\left(-\frac{bx}{2}-\frac{a}{2}+\frac{1}{2}\right)\ln\left(\frac{bx}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2\,b^3}+\frac{a\ln\left(-\frac{bx}{2}-\frac{a}{2}+\frac{1}{2}\right)\ln(bx+a+1)}{2\,b^3}-\frac{a^2\ln\left(-\frac{bx}{2}-\frac{a}{2}+\frac{1}{2}\right)\ln\left(\frac{bx}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2\,b^3}$$
$$+\frac{a^2\ln\left(-\frac{bx}{2}-\frac{a}{2}+\frac{1}{2}\right)\ln(bx+a+1)}{2\,b^3}-\frac{4\,\text{arccoth}(bx+a)\,xa}{3\,b^2}$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int (fx+e)^3\,(a+b\,\text{arccoth}(dx+c))\,dx$$

Optimal(type 3, 156 leaves, 7 steps):

$$\frac{bf\left(6\,d^2e^2-12\,cdef+\left(6\,c^2+1\right)f^2\right)x}{4\,d^3}+\frac{bf^2\,(-fc+de)\,(dx+c)^2}{2\,d^4}+\frac{bf^3\,(dx+c)^3}{12\,d^4}+\frac{(fx+e)^4\,(a+b\,\text{arccoth}(dx+c))}{4\,f}$$
$$+\frac{b\,(-fc+de+f)^4\ln(-dx-c+1)}{8\,d^4f}-\frac{b\,(-fc+de-f)^4\ln(dx+c+1)}{8\,d^4f}$$

Result(type 3, 785 leaves):

$$\frac{af^3x^4}{4}+axe^3+\frac{ae^4}{4f}+\frac{13\,bf^3c^3}{12\,d^4}+\frac{bf^3c}{4\,d^4}+\frac{bf^3x^3}{12\,d}+\frac{bf^3\ln(dx+c-1)}{8\,d^4}-\frac{bf^3\ln(dx+c+1)}{8\,d^4}+\frac{b\ln(dx+c-1)\,e^3}{2\,d}+\frac{b\ln(dx+c+1)\,e^3}{2\,d}$$
$$+\frac{bf^3\,\text{arccoth}(dx+c)\,x^4}{4}+\text{arccoth}(dx+c)\,xbe^3+\frac{b\ln(dx+c-1)\,e^4}{8f}-\frac{b\ln(dx+c+1)\,e^4}{8f}+\frac{b\,\text{arccoth}(dx+c)\,e^4}{4f}+\frac{bf^3x}{4\,d^3}+af^2x^3e$$
$$+\frac{3\,afx^2e^2}{2}+\frac{b\ln(dx+c+1)\,ce^3}{2\,d}+\frac{bf^2\ln(dx+c+1)\,e}{2\,d^3}+\frac{3\,bf^3\ln(dx+c-1)\,c^2}{4\,d^4}+bf^2\,\text{arccoth}(dx+c)\,ex^3+\frac{3\,bf\,\text{arccoth}(dx+c)\,e^2x^2}{2}$$
$$+\frac{3\,bf^3c^2x}{4\,d^3}+\frac{3\,bfe^2x}{2\,d}+\frac{3\,bfe^2c}{2\,d^2}-\frac{5\,bf^2ec^2}{2\,d^3}-\frac{bf^3x^2c}{4\,d^2}-\frac{bf^3\ln(dx+c-1)\,c}{2\,d^4}-\frac{bf^3\ln(dx+c+1)\,c^4}{8\,d^4}-\frac{bf^3\ln(dx+c+1)\,c^3}{2\,d^4}$$

$$-\frac{3bf^3\ln(dx+c+1)c^2}{4d^4}-\frac{bf^3\ln(dx+c+1)c}{2d^4}+\frac{3bf\ln(dx+c-1)e^2}{4d^2}+\frac{bf^2\ln(dx+c-1)e}{2d^3}-\frac{3bf\ln(dx+c+1)e^2}{4d^2}$$
$$+\frac{bf^3\ln(dx+c-1)c^4}{8d^4}-\frac{bf^3\ln(dx+c-1)c^3}{2d^4}-\frac{b\ln(dx+c-1)ce^3}{2d}+\frac{bf^2ex^2}{2d}-\frac{3bf\ln(dx+c+1)ce^2}{2d^2}+\frac{3bf^2\ln(dx+c+1)ce}{2d^3}$$
$$-\frac{bf^2\ln(dx+c-1)c^3e}{2d^3}+\frac{3bf\ln(dx+c-1)c^2e^2}{4d^2}+\frac{3bf^2\ln(dx+c-1)c^2e}{2d^3}-\frac{3bf\ln(dx+c-1)ce^2}{2d^2}-\frac{3bf^2\ln(dx+c-1)ce}{2d^3}$$
$$+\frac{bf^2\ln(dx+c+1)c^3e}{2d^3}-\frac{3bf\ln(dx+c+1)c^2e^2}{4d^2}+\frac{3bf^2\ln(dx+c+1)c^2e}{2d^3}-\frac{2bf^2cex}{d^2}$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int\frac{(a+b\operatorname{arccoth}(dx+c))^2}{(fx+e)^2}\,dx$$

Optimal(type 4, 478 leaves, 24 steps):

$$-\frac{(a+b\operatorname{arccoth}(dx+c))^2}{f(fx+e)}+\frac{b^2d\operatorname{arccoth}(dx+c)\ln\left(\frac{2}{-dx-c+1}\right)}{f(-fc+de+f)}-\frac{abd\ln(-dx-c+1)}{f(-fc+de+f)}-\frac{b^2d\operatorname{arccoth}(dx+c)\ln\left(\frac{2}{dx+c+1}\right)}{f(-fc+de-f)}$$
$$+\frac{2b^2d\operatorname{arccoth}(dx+c)\ln\left(\frac{2}{dx+c+1}\right)}{(-fc+de+f)(de-(1+c)f)}+\frac{abd\ln(dx+c+1)}{f(-fc+de-f)}+\frac{2abd\ln(fx+e)}{f^2-(-fc+de)^2}-\frac{2b^2d\operatorname{arccoth}(dx+c)\ln\left(\frac{2d(fx+e)}{(-fc+de+f)(dx+c+1)}\right)}{(-fc+de+f)(de-(1+c)f)}$$
$$+\frac{b^2d\operatorname{polylog}\left(2,\frac{-dx-c-1}{-dx-c+1}\right)}{2f(-fc+de+f)}+\frac{b^2d\operatorname{polylog}\left(2,1-\frac{2}{dx+c+1}\right)}{2f(-fc+de-f)}-\frac{b^2d\operatorname{polylog}\left(2,1-\frac{2}{dx+c+1}\right)}{(-fc+de+f)(de-(1+c)f)}$$
$$+\frac{b^2d\operatorname{polylog}\left(2,1-\frac{2d(fx+e)}{(-fc+de+f)(dx+c+1)}\right)}{(-fc+de+f)(de-(1+c)f)}$$

Result(type 4, 1285 leaves):

$$-\frac{db^2\ln(dx+c+1)^2}{4(fc-de-f)(fc-de+f)}-\frac{db^2\operatorname{dilog}\left(\frac{(dx+c)f-f}{fc-de-f}\right)}{(fc-de-f)(fc-de+f)}+\frac{db^2\operatorname{dilog}\left(\frac{(dx+c)f+f}{fc-de+f}\right)}{(fc-de-f)(fc-de+f)}+\frac{db^2\ln(dx+c-1)^2}{4(fc-de-f)(fc-de+f)}$$
$$-\frac{db^2\operatorname{dilog}\left(\frac{dx}{2}+\frac{c}{2}+\frac{1}{2}\right)}{(fc-de-f)(fc-de+f)}-\frac{db^2\operatorname{arccoth}(dx+c)^2}{(dfx+de)f}-\frac{db^2c\ln(dx+c-1)\ln\left(\frac{dx}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2(fc-de-f)(fc-de+f)}$$
$$+\frac{db^2c\ln\left(-\frac{dx}{2}-\frac{c}{2}+\frac{1}{2}\right)\ln\left(\frac{dx}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2(fc-de-f)(fc-de+f)}-\frac{db^2c\ln\left(-\frac{dx}{2}-\frac{c}{2}+\frac{1}{2}\right)\ln(dx+c+1)}{2(fc-de-f)(fc-de+f)}-\frac{d^2b^2e\ln(dx+c+1)^2}{4f(fc-de-f)(fc-de+f)}$$
$$-\frac{d^2b^2e\ln(dx+c-1)^2}{4f(fc-de-f)(fc-de+f)}+\frac{db^2c\ln(dx+c-1)^2}{4(fc-de-f)(fc-de+f)}+\frac{db^2c\ln(dx+c+1)^2}{4(fc-de-f)(fc-de+f)}-\frac{2dab\ln(dx+c+1)}{f(2fc-2de+2f)}$$

$$+\frac{2 d a b \ln(d x+c-1)}{f(2fc-2de-2f)}-\frac{2 d a b \ln((d x+c) f-fc+de)}{(fc-de-f)(fc-de+f)}-\frac{2 d b^2 \operatorname{arccoth}(d x+c) \ln(d x+c+1)}{f(2fc-2de+2f)}+\frac{2 d b^2 \operatorname{arccoth}(d x+c) \ln(d x+c-1)}{f(2fc-2de-2f)}$$

$$-\frac{2 d b^2 \operatorname{arccoth}(d x+c) \ln((d x+c) f-fc+de)}{(fc-de-f)(fc-de+f)}-\frac{d b^2 \ln((d x+c) f-fc+de) \ln\left(\frac{(d x+c) f-f}{fc-de-f}\right)}{(fc-de-f)(fc-de+f)}$$

$$+\frac{d b^2 \ln((d x+c) f-fc+de) \ln\left(\frac{(d x+c) f+f}{fc-de+f}\right)}{(fc-de-f)(fc-de+f)}-\frac{d b^2 \ln(d x+c-1) \ln\left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2(fc-de-f)(fc-de+f)}-\frac{d b^2 \ln\left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln\left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2(fc-de-f)(fc-de+f)}$$

$$+\frac{d b^2 \ln\left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln(d x+c+1)}{2(fc-de-f)(fc-de+f)}-\frac{2 d a b \operatorname{arccoth}(d x+c)}{(d f x+d e) f}-\frac{d a^2}{(d f x+d e) f}+\frac{d^2 b^2 e \ln\left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln(d x+c+1)}{2 f(fc-de-f)(fc-de+f)}$$

$$+\frac{d^2 b^2 e \ln(d x+c-1) \ln\left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2 f(fc-de-f)(fc-de+f)}-\frac{d^2 b^2 e \ln\left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln\left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2 f(fc-de-f)(fc-de+f)}$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \operatorname{arccoth}(d x+c))^3}{f x+e}\, dx$$

Optimal(type 4, 296 leaves, 2 steps):

$$-\frac{(a+b \operatorname{arccoth}(d x+c))^3 \ln\left(\frac{2}{d x+c+1}\right)}{f}+\frac{(a+b \operatorname{arccoth}(d x+c))^3 \ln\left(\frac{2 d (f x+e)}{(-fc+de+f)(d x+c+1)}\right)}{f}$$

$$+\frac{3 b (a+b \operatorname{arccoth}(d x+c))^2 \operatorname{polylog}\left(2, 1-\frac{2}{d x+c+1}\right)}{2 f}-\frac{3 b (a+b \operatorname{arccoth}(d x+c))^2 \operatorname{polylog}\left(2, 1-\frac{2 d (f x+e)}{(-fc+de+f)(d x+c+1)}\right)}{2 f}$$

$$+\frac{3 b^2 (a+b \operatorname{arccoth}(d x+c)) \operatorname{polylog}\left(3, 1-\frac{2}{d x+c+1}\right)}{2 f}-\frac{3 b^2 (a+b \operatorname{arccoth}(d x+c)) \operatorname{polylog}\left(3, 1-\frac{2 d (f x+e)}{(-fc+de+f)(d x+c+1)}\right)}{2 f}$$

$$+\frac{3 b^3 \operatorname{polylog}\left(4, 1-\frac{2}{d x+c+1}\right)}{4 f}-\frac{3 b^3 \operatorname{polylog}\left(4, 1-\frac{2 d (f x+e)}{(-fc+de+f)(d x+c+1)}\right)}{4 f}$$

Result(type ?, 3795 leaves): Display of huge result suppressed!

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \operatorname{arccoth}(d x+c))^3}{(f x+e)^2}\, dx$$

Optimal(type 4, 1067 leaves, 33 steps):

$$-\frac{(a+b \operatorname{arccoth}(d x+c))^3}{f(f x+e)}+\frac{3 a b^2 d \operatorname{arccoth}(d x+c) \ln\left(\frac{2}{-d x-c+1}\right)}{f(-fc+de+f)}+\frac{3 b^3 d \operatorname{arccoth}(d x+c)^2 \ln\left(\frac{2}{-d x-c+1}\right)}{2 f(-fc+de+f)}-\frac{3 a^2 b d \ln(-d x-c+1)}{2 f(-fc+de+f)}$$

$$- \frac{3 a b^2 d \operatorname{arccoth}(d x + c) \ln\left(\frac{2}{d x + c + 1}\right)}{f(-fc + d e - f)} + \frac{6 a b^2 d \operatorname{arccoth}(d x + c) \ln\left(\frac{2}{d x + c + 1}\right)}{(-fc + d e + f) (d e - (1 + c) f)} - \frac{3 b^3 d \operatorname{arccoth}(d x + c)^2 \ln\left(\frac{2}{d x + c + 1}\right)}{2 f(-fc + d e - f)}$$

$$+ \frac{3 b^3 d \operatorname{arccoth}(d x + c)^2 \ln\left(\frac{2}{d x + c + 1}\right)}{(-fc + d e + f) (d e - (1 + c) f)} + \frac{3 a^2 b d \ln(d x + c + 1)}{2 f(-fc + d e - f)} + \frac{3 a^2 b d \ln(fx + e)}{f^2 - (-fc + d e)^2}$$

$$- \frac{6 a b^2 d \operatorname{arccoth}(d x + c) \ln\left(\frac{2 d (fx + e)}{(-fc + d e + f) (d x + c + 1)}\right)}{(-fc + d e + f) (d e - (1 + c) f)} - \frac{3 b^3 d \operatorname{arccoth}(d x + c)^2 \ln\left(\frac{2 d (fx + e)}{(-fc + d e + f) (d x + c + 1)}\right)}{(-fc + d e + f) (d e - (1 + c) f)}$$

$$+ \frac{3 a b^2 d \operatorname{polylog}\left(2, \frac{-d x - c - 1}{-d x - c + 1}\right)}{2 f(-fc + d e + f)} + \frac{3 b^3 d \operatorname{arccoth}(d x + c) \operatorname{polylog}\left(2, 1 - \frac{2}{-d x - c + 1}\right)}{2 f(-fc + d e + f)} + \frac{3 a b^2 d \operatorname{polylog}\left(2, 1 - \frac{2}{d x + c + 1}\right)}{2 f(-fc + d e - f)}$$

$$- \frac{3 a b^2 d \operatorname{polylog}\left(2, 1 - \frac{2}{d x + c + 1}\right)}{(-fc + d e + f) (d e - (1 + c) f)} + \frac{3 b^3 d \operatorname{arccoth}(d x + c) \operatorname{polylog}\left(2, 1 - \frac{2}{d x + c + 1}\right)}{2 f(-fc + d e - f)} - \frac{3 b^3 d \operatorname{arccoth}(d x + c) \operatorname{polylog}\left(2, 1 - \frac{2}{d x + c + 1}\right)}{(-fc + d e + f) (d e - (1 + c) f)}$$

$$+ \frac{3 a b^2 d \operatorname{polylog}\left(2, 1 - \frac{2 d (fx + e)}{(-fc + d e + f) (d x + c + 1)}\right)}{(-fc + d e + f) (d e - (1 + c) f)} + \frac{3 b^3 d \operatorname{arccoth}(d x + c) \operatorname{polylog}\left(2, 1 - \frac{2 d (fx + e)}{(-fc + d e + f) (d x + c + 1)}\right)}{(-fc + d e + f) (d e - (1 + c) f)}$$

$$- \frac{3 b^3 d \operatorname{polylog}\left(3, 1 - \frac{2}{-d x - c + 1}\right)}{4 f(-fc + d e + f)} + \frac{3 b^3 d \operatorname{polylog}\left(3, 1 - \frac{2}{d x + c + 1}\right)}{4 f(-fc + d e - f)} - \frac{3 b^3 d \operatorname{polylog}\left(3, 1 - \frac{2}{d x + c + 1}\right)}{2 (-fc + d e + f) (d e - (1 + c) f)}$$

$$+ \frac{3 b^3 d \operatorname{polylog}\left(3, 1 - \frac{2 d (fx + e)}{(-fc + d e + f) (d x + c + 1)}\right)}{2 (-fc + d e + f) (d e - (1 + c) f)}$$

Result(type ?, 5130 leaves): Display of huge result suppressed!

Problem 34: Unable to integrate problem.

$$\int (fx + e)^m (a + b \operatorname{arccoth}(d x + c)) \, dx$$

Optimal(type 5, 162 leaves, 6 steps):

$$\frac{(fx + e)^{1 + m} (a + b \operatorname{arccoth}(d x + c))}{f(1 + m)} + \frac{b d (fx + e)^{2 + m} \operatorname{hypergeom}\left([1, 2 + m], [3 + m], \frac{d (fx + e)}{-fc + d e - f}\right)}{2 f(d e - (1 + c) f) (1 + m) (2 + m)}$$

$$- \frac{b d (fx + e)^{2 + m} \operatorname{hypergeom}\left([1, 2 + m], [3 + m], \frac{d (fx + e)}{-fc + d e + f}\right)}{2 f(-fc + d e + f) (1 + m) (2 + m)}$$

Result(type 8, 20 leaves):

$$\int (fx + e)^m (a + b \operatorname{arccoth}(d x + c)) \, dx$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a + b\,\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}{-c^2 x^2 + 1}\, dx$$

Optimal(type 4, 258 leaves, 7 steps):

$$-\frac{2\,\mathrm{arccoth}\left(1 - \frac{2}{1 - \frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}\right)\left(a + b\,\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}{c} - \frac{b\left(a + b\,\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)\mathrm{polylog}\left(2, 1 - \frac{2}{1 + \frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}\right)}{c}$$
$$+ \frac{b\left(a + b\,\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)\mathrm{polylog}\left(2, 1 - \frac{2\sqrt{-cx+1}}{\left(1 + \frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\sqrt{cx+1}}\right)}{c} - \frac{b^2\,\mathrm{polylog}\left(3, 1 - \frac{2}{1 + \frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}\right)}{2c}$$
$$+ \frac{b^2\,\mathrm{polylog}\left(3, 1 - \frac{2\sqrt{-cx+1}}{\left(1 + \frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\sqrt{cx+1}}\right)}{2c}$$

Result(type 4, 695 leaves):

$$-\frac{a^2 \ln(cx-1)}{2c} + \frac{a^2 \ln(cx+1)}{2c} - \frac{b^2\,\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(1 + \frac{1 + \frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} - 1}\right)}{c} - \frac{b^2\,\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\mathrm{polylog}\left(2, -\frac{1 + \frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} - 1}\right)}{c}$$
$$+ \frac{b^2\,\mathrm{polylog}\left(3, -\frac{1 + \frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} - 1}\right)}{2c} + \frac{b^2\,\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(1 - \frac{1}{\sqrt{\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} - 1}{1 + \frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}}}\right)}{c}$$

$$+\frac{2 b^2 \operatorname{arccoth}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right) \operatorname{polylog}\left(2, \frac{1}{\sqrt{\frac{\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}-1}{1+\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}}}}\right)}{c}-\frac{2 b^2 \operatorname{polylog}\left(3, \frac{1}{\sqrt{\frac{\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}-1}{1+\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}}}}\right)}{c}$$

$$+\frac{b^2 \operatorname{arccoth}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)^2 \ln\left(1+\frac{1}{\sqrt{\frac{\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}-1}{1+\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}}}}\right)}{c}+\frac{2 b^2 \operatorname{arccoth}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right) \operatorname{polylog}\left(2, -\frac{1}{\sqrt{\frac{\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}-1}{1+\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}}}}\right)}{c}$$

$$-\frac{2 b^2 \operatorname{polylog}\left(3, -\frac{1}{\sqrt{\frac{\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}-1}{1+\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}}}}\right)}{c}+\frac{2 a b \operatorname{dilog}\left(\frac{\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}-1}{1+\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}}\right)}{c}-\frac{a b \operatorname{dilog}\left(\frac{\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}-1\right)^2}{\left(1+\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)^2}\right)}{2 c}$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \operatorname{arccoth}(\tanh(b x+a))\, dx$$

Optimal(type 3, 14 leaves, 2 steps):

$$\frac{\operatorname{arccoth}(\tanh(b x+a))^2}{2 b}$$

Result(type 3, 31 leaves):

$$\frac{\operatorname{arctanh}(\tanh(b x+a)) \operatorname{arccoth}(\tanh(b x+a))-\frac{\operatorname{arctanh}(\tanh(b x+a))^2}{2}}{b}$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{arccoth}(\tanh(b x+a))^2}{x^2}\, dx$$

Optimal(type 3, 39 leaves, 3 steps):

$$2b^2x-\frac{\operatorname{arccoth}(\tanh(bx+a))^2}{x}-2b\,(bx-\operatorname{arccoth}(\tanh(bx+a)))\ln(x)$$

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Result(type 3, 1098 leaves):
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$$2b^2x+I\pi\ln(x)\,b\operatorname{csgn}(I e^{bx+a})\operatorname{csgn}(I e^{2bx+2a})^2-\frac{I\pi\ln(e^{bx+a})\operatorname{csgn}(I e^{2bx+2a})\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2}{2x}$$

$$-\frac{I\pi\ln(e^{bx+a})\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2}{2x}+\frac{I\pi\ln(e^{bx+a})\operatorname{csgn}(I e^{bx+a})^2\operatorname{csgn}(I e^{2bx+2a})}{2x}$$

$$-\frac{I\pi\ln(e^{bx+a})\operatorname{csgn}(I e^{bx+a})\operatorname{csgn}(I e^{2bx+2a})^2}{x}+\frac{I\pi\ln(e^{bx+a})\operatorname{csgn}(I e^{2bx+2a})^3}{2x}+\frac{I\pi\ln(e^{bx+a})\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}+1}\right)^3}{2x}$$

$$-\frac{I\pi\ln(x)\,b\operatorname{csgn}(I e^{2bx+2a})^3}{2}-\frac{I\pi\ln(x)\,b\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}+1}\right)^3}{2}+\frac{I\pi\ln(x)\,b\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2}{2}$$

$$-\frac{I\pi\ln(x)\,b\operatorname{csgn}(I e^{bx+a})^2\operatorname{csgn}(I e^{2bx+2a})}{2}+\frac{I\pi\ln(x)\,b\operatorname{csgn}(I e^{2bx+2a})\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2}{2}+\frac{I\pi\ln(e^{bx+a})\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}+1}\right)^3}{x}$$

$$-I\pi\ln(x)\,b\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}+1}\right)^3+I\pi\ln(x)\,b\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}+1}\right)^2-\frac{I\pi\ln(e^{bx+a})\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}+1}\right)^2}{x}-2\ln(x)\,x b^2+2\ln(e^{bx+a})\ln(x)\,b$$

$$-I\pi\ln(x)\,b+\frac{I\pi\ln(e^{bx+a})}{x}-\frac{\ln(e^{bx+a})^2}{x}-\frac{I\pi\ln(x)\,b\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}+1}\right)\operatorname{csgn}(I e^{2bx+2a})\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}+1}\right)}{2}$$

$$+\frac{I\pi\ln(e^{bx+a})\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}+1}\right)\operatorname{csgn}(I e^{2bx+2a})\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}+1}\right)}{2x}+\frac{1}{16x}\Bigg(\pi^2\Bigg(2\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}+1}\right)^2$$

$$-2\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}+1}\right)^3-\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}+1}\right)\operatorname{csgn}(I e^{2bx+2a})\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}+1}\right)+\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2$$

$$-\operatorname{csgn}(I e^{bx+a})^2\operatorname{csgn}(I e^{2bx+2a})+2\operatorname{csgn}(I e^{bx+a})\operatorname{csgn}(I e^{2bx+2a})^2-\operatorname{csgn}(I e^{2bx+2a})^3+\operatorname{csgn}(I e^{2bx+2a})\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2$$

$$-\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}+1}\right)^3-2\Bigg)^2\Bigg)$$

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Problem 42: Result more than twice size of optimal antiderivative.
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$$\int x^4 \operatorname{arccoth}(\tanh(bx+a))^3\, dx$$

Optimal(type 3, 53 leaves, 4 steps):

$$-\frac{b^3 x^8}{280} + \frac{b^2 x^7 \operatorname{arccoth}(\tanh(bx+a))}{35} - \frac{b x^6 \operatorname{arccoth}(\tanh(bx+a))^2}{10} + \frac{x^5 \operatorname{arccoth}(\tanh(bx+a))^3}{5}$$

Result(type ?, 18110 leaves): Display of huge result suppressed!

Problem 43: Result more than twice size of optimal antiderivative.

$$\int x^3 \operatorname{arccoth}(\tanh(bx+a))^3\, dx$$

Optimal(type 3, 53 leaves, 4 steps):

$$-\frac{b^3 x^7}{140} + \frac{b^2 x^6 \operatorname{arccoth}(\tanh(bx+a))}{20} - \frac{3 b x^5 \operatorname{arccoth}(\tanh(bx+a))^2}{20} + \frac{x^4 \operatorname{arccoth}(\tanh(bx+a))^3}{4}$$

Result(type ?, 18110 leaves): Display of huge result suppressed!

Problem 44: Humongous result has more than 20000 leaves.

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^3}{x}\, dx$$

Optimal(type 3, 73 leaves, 4 steps):

$$b x (b x - \operatorname{arccoth}(\tanh(bx+a)))^2 - \frac{(bx - \operatorname{arccoth}(\tanh(bx+a)))\operatorname{arccoth}(\tanh(bx+a))^2}{2} + \frac{\operatorname{arccoth}(\tanh(bx+a))^3}{3} - (bx - \operatorname{arccoth}(\tanh(bx+a)))^3 \ln(x)$$

Result(type ?, 21847 leaves): Display of huge result suppressed!

Problem 45: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^3}{x^4}\, dx$$

Optimal(type 3, 51 leaves, 4 steps):

$$-\frac{b^2 \operatorname{arccoth}(\tanh(bx+a))}{x} - \frac{b \operatorname{arccoth}(\tanh(bx+a))^2}{2 x^2} - \frac{\operatorname{arccoth}(\tanh(bx+a))^3}{3 x^3} + b^3 \ln(x)$$

Result(type ?, 17236 leaves): Display of huge result suppressed!

Problem 46: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^3}{x^5}\, dx$$

Optimal(type 3, 29 leaves, 1 step):

$$\frac{\operatorname{arccoth}(\tanh(bx+a))^4}{4 x^4 (bx - \operatorname{arccoth}(\tanh(bx+a)))}$$

Result(type ?, 17234 leaves): Display of huge result suppressed!

Problem 47: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx+a))}\, dx$$

Optimal(type 3, 90 leaves, 6 steps):

$$\frac{b}{x\,(bx-\operatorname{arccoth}(\tanh(bx+a)))^2}+\frac{1}{2x^2\,(bx-\operatorname{arccoth}(\tanh(bx+a)))}-\frac{b^2\ln(x)}{(bx-\operatorname{arccoth}(\tanh(bx+a)))^3}+\frac{b^2\ln(\operatorname{arccoth}(\tanh(bx+a)))}{(bx-\operatorname{arccoth}(\tanh(bx+a)))^3}$$

Result(type 1, 1 leaves):???

Problem 48: Humongous result has more than 20000 leaves.

$$\int \frac{x^4}{\operatorname{arccoth}(\tanh(bx+a))^2}\, dx$$

Optimal(type 3, 96 leaves, 6 steps):

$$\frac{4x^3}{3b^2}+\frac{2x^2\,(bx-\operatorname{arccoth}(\tanh(bx+a)))}{b^3}+\frac{4x\,(bx-\operatorname{arccoth}(\tanh(bx+a)))^2}{b^4}-\frac{x^4}{b\operatorname{arccoth}(\tanh(bx+a))}$$
$$+\frac{4\,(bx-\operatorname{arccoth}(\tanh(bx+a)))^3\ln(\operatorname{arccoth}(\tanh(bx+a)))}{b^5}$$

Result(type ?, 131084 leaves): Display of huge result suppressed!

Problem 49: Humongous result has more than 20000 leaves.

$$\int x \operatorname{arccoth}(\tanh(bx+a))^n\, dx$$

Optimal(type 3, 48 leaves, 3 steps):

$$\frac{x\operatorname{arccoth}(\tanh(bx+a))^{1+n}}{b\,(1+n)}-\frac{\operatorname{arccoth}(\tanh(bx+a))^{2+n}}{b^2\,(1+n)\,(2+n)}$$

Result(type ?, 71610 leaves): Display of huge result suppressed!

Problem 50: Unable to integrate problem.

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^n}{x}\, dx$$

Optimal(type 5, 66 leaves, 1 step):

$$\frac{\operatorname{arccoth}(\tanh(bx+a))^{1+n}\operatorname{hypergeom}\left([1, 1+n], [2+n], -\frac{\operatorname{arccoth}(\tanh(bx+a))}{bx-\operatorname{arccoth}(\tanh(bx+a))}\right)}{(1+n)\,(bx-\operatorname{arccoth}(\tanh(bx+a)))}$$

Result(type 8, 15 leaves):

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^n}{x}\, dx$$

Problem 52: Unable to integrate problem.

$$\int x \operatorname{arccoth}(\sinh(x))\, dx$$

Optimal(type 1, 1 leaves, 8 steps):

$$0$$

Result(type 8, 7 leaves):

$$\int x \operatorname{arccoth}(\sinh(x))\, dx$$

Problem 53: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{arccoth}(c + d\tanh(bx + a))\, dx$$

Optimal(type 4, 211 leaves, 9 steps):

$$\frac{x^2 \operatorname{arccoth}(c + d\tanh(bx+a))}{2} + \frac{x^2 \ln\left(1 + \frac{(1-c-d)\, e^{2bx+2a}}{1-c+d}\right)}{4} - \frac{x^2 \ln\left(1 + \frac{(1+c+d)\, e^{2bx+2a}}{1+c-d}\right)}{4} + \frac{x \operatorname{polylog}\left(2, -\frac{(1-c-d)\, e^{2bx+2a}}{1-c+d}\right)}{4b} - \frac{x \operatorname{polylog}\left(2, -\frac{(1+c+d)\, e^{2bx+2a}}{1+c-d}\right)}{4b} - \frac{\operatorname{polylog}\left(3, -\frac{(1-c-d)\, e^{2bx+2a}}{1-c+d}\right)}{8b^2} + \frac{\operatorname{polylog}\left(3, -\frac{(1+c+d)\, e^{2bx+2a}}{1+c-d}\right)}{8b^2}$$

Result(type ?, 4989 leaves): Display of huge result suppressed!

Problem 55: Result more than twice size of optimal antiderivative.

$$\int x^3 \operatorname{arccoth}(1 + d + d\tanh(bx + a))\, dx$$

Optimal(type 4, 136 leaves, 8 steps):

$$\frac{b x^5}{20} + \frac{x^4 \operatorname{arccoth}(1+d+d\tanh(bx+a))}{4} - \frac{x^4 \ln\left(1 + (1+d)\, e^{2bx+2a}\right)}{8} - \frac{x^3 \operatorname{polylog}\left(2, -(1+d)\, e^{2bx+2a}\right)}{4b} + \frac{3x^2 \operatorname{polylog}\left(3, -(1+d)\, e^{2bx+2a}\right)}{8b^2} - \frac{3x \operatorname{polylog}\left(4, -(1+d)\, e^{2bx+2a}\right)}{8b^3} + \frac{3 \operatorname{polylog}\left(5, -(1+d)\, e^{2bx+2a}\right)}{16b^4}$$

Result(type 4, 1735 leaves):

$$\frac{b x^5}{20} + \frac{a^4 \ln\left(1 - e^{bx+a}\sqrt{-d-1}\right)}{2b^4(1+d)} + \frac{a^4 \ln\left(1 + e^{bx+a}\sqrt{-d-1}\right)}{2b^4(1+d)} + \frac{a^3 \operatorname{dilog}\left(1 + e^{bx+a}\sqrt{-d-1}\right)}{2b^4(1+d)} + \frac{a^3 \operatorname{dilog}\left(1 - e^{bx+a}\sqrt{-d-1}\right)}{2b^4(1+d)} - \frac{3\ln\left(1 + (1+d)\, e^{2bx+2a}\right) a^4}{8b^4(1+d)} - \frac{d \ln\left(1 + (1+d)\, e^{2bx+2a}\right) x^4}{8(1+d)} + \frac{x^4 \ln\left(e^{2bx+2a} d + e^{2bx+2a} + 1\right)}{8} - \frac{x^4 \ln\left(e^{bx+a}\right)}{4} - \frac{\ln(d)\, x^4}{8} + \frac{d a^3 \operatorname{dilog}\left(1 + e^{bx+a}\sqrt{-d-1}\right)}{2b^4(1+d)} + \frac{d a^3 \operatorname{dilog}\left(1 - e^{bx+a}\sqrt{-d-1}\right)}{2b^4(1+d)} - \frac{\ln\left(1 + (1+d)\, e^{2bx+2a}\right) x a^3}{2b^3(1+d)} - \frac{3 d a^4 \ln\left(1 + (1+d)\, e^{2bx+2a}\right)}{8b^4(1+d)} + \frac{d a^4 \ln\left(1 + e^{bx+a}\sqrt{-d-1}\right)}{2b^4(1+d)} + \frac{d a^4 \ln\left(1 - e^{bx+a}\sqrt{-d-1}\right)}{2b^4(1+d)} + \frac{a^3 \ln\left(1 - e^{bx+a}\sqrt{-d-1}\right) x}{2b^3(1+d)} - \frac{\ln\left(1 + (1+d)\, e^{2bx+2a}\right) x^4}{8(1+d)}$$

$$- \frac{I \pi \operatorname{csgn}\left(\frac{I}{e^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(I\left(e^{2 b x+2 a} d+e^{2 b x+2 a}+1\right)\right) \operatorname{csgn}\left(\frac{I\left(e^{2 b x+2 a} d+e^{2 b x+2 a}+1\right)}{e^{2 b x+2 a}+1}\right) x^4}{16}$$

$$+ \frac{I \pi \operatorname{csgn}\left(\frac{I}{e^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(I e^{2 b x+2 a}\right) \operatorname{csgn}\left(\frac{I e^{2 b x+2 a}}{e^{2 b x+2 a}+1}\right) x^4}{16} + \frac{d a^3 \ln\left(1+e^{b x+a} \sqrt{-d-1}\right) x}{2 b^3 (1+d)} + \frac{d a^3 \ln\left(1-e^{b x+a} \sqrt{-d-1}\right) x}{2 b^3 (1+d)}$$

$$- \frac{d a^3 \ln\left(1+(1+d) e^{2 b x+2 a}\right) x}{2 b^3 (1+d)} + \frac{I \pi \operatorname{csgn}\left(\frac{I d e^{2 b x+2 a}}{e^{2 b x+2 a}+1}\right)^3 x^4}{16} + \frac{I \pi \operatorname{csgn}\left(I e^{2 b x+2 a}\right)^3 x^4}{16} + \frac{I \pi \operatorname{csgn}\left(\frac{I e^{2 b x+2 a}}{e^{2 b x+2 a}+1}\right)^3 x^4}{16}$$

$$- \frac{I \pi \operatorname{csgn}\left(\frac{I\left(e^{2 b x+2 a} d+e^{2 b x+2 a}+1\right)}{e^{2 b x+2 a}+1}\right)^3 x^4}{16} - \frac{3 d \operatorname{polylog}\left(4, (-d-1) e^{2 b x+2 a}\right) x}{8 b^3 (1+d)} - \frac{d a^3 \operatorname{polylog}\left(2, (-d-1) e^{2 b x+2 a}\right)}{4 b^4 (1+d)}$$

$$- \frac{d a^4 \ln\left(e^{2 b x+2 a} d+e^{2 b x+2 a}+1\right)}{8 b^4 (1+d)} - \frac{d \operatorname{polylog}\left(2, (-d-1) e^{2 b x+2 a}\right) x^3}{4 b (1+d)} + \frac{3 d \operatorname{polylog}\left(3, (-d-1) e^{2 b x+2 a}\right) x^2}{8 b^2 (1+d)} + \frac{a^3 \ln\left(1+e^{b x+a} \sqrt{-d-1}\right) x}{2 b^3 (1+d)}$$

$$- \frac{I \pi \operatorname{csgn}\left(\frac{I e^{2 b x+2 a}}{e^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\frac{I d e^{2 b x+2 a}}{e^{2 b x+2 a}+1}\right)^2 x^4}{16} - \frac{I \pi \operatorname{csgn}\left(I e^{b x+a}\right) \operatorname{csgn}\left(I e^{2 b x+2 a}\right)^2 x^4}{8} - \frac{I \pi \operatorname{csgn}\left(I e^{2 b x+2 a}\right) \operatorname{csgn}\left(\frac{I e^{2 b x+2 a}}{e^{2 b x+2 a}+1}\right)^2 x^4}{16}$$

$$- \frac{I \pi \operatorname{csgn}\left(\frac{I}{e^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\frac{I e^{2 b x+2 a}}{e^{2 b x+2 a}+1}\right)^2 x^4}{16} + \frac{I \pi \operatorname{csgn}\left(\frac{I}{e^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\frac{I\left(e^{2 b x+2 a} d+e^{2 b x+2 a}+1\right)}{e^{2 b x+2 a}+1}\right)^2 x^4}{16}$$

$$+ \frac{I \pi \operatorname{csgn}\left(I e^{b x+a}\right)^2 \operatorname{csgn}\left(I e^{2 b x+2 a}\right) x^4}{16} + \frac{I \pi \operatorname{csgn}\left(I\left(e^{2 b x+2 a} d+e^{2 b x+2 a}+1\right)\right) \operatorname{csgn}\left(\frac{I\left(e^{2 b x+2 a} d+e^{2 b x+2 a}+1\right)}{e^{2 b x+2 a}+1}\right)^2 x^4}{16}$$

$$- \frac{\operatorname{polylog}\left(2, (-d-1) e^{2 b x+2 a}\right) x^3}{4 b (1+d)} - \frac{\operatorname{polylog}\left(2, (-d-1) e^{2 b x+2 a}\right) a^3}{4 b^4 (1+d)} + \frac{3 \operatorname{polylog}\left(3, (-d-1) e^{2 b x+2 a}\right) x^2}{8 b^2 (1+d)} - \frac{a^4 \ln\left(e^{2 b x+2 a} d+e^{2 b x+2 a}+1\right)}{8 b^4 (1+d)}$$

$$+ \frac{3 \operatorname{polylog}\left(5, (-d-1) e^{2 b x+2 a}\right)}{16 b^4 (1+d)} + \frac{3 d \operatorname{polylog}\left(5, (-d-1) e^{2 b x+2 a}\right)}{16 b^4 (1+d)} - \frac{3 \operatorname{polylog}\left(4, (-d-1) e^{2 b x+2 a}\right) x}{8 b^3 (1+d)}$$

$$- \frac{I \pi \operatorname{csgn}(I d) \operatorname{csgn}\left(\frac{I d e^{2 b x+2 a}}{e^{2 b x+2 a}+1}\right)^2 x^4}{16} + \frac{I \pi \operatorname{csgn}\left(\frac{I e^{2 b x+2 a}}{e^{2 b x+2 a}+1}\right) \operatorname{csgn}(I d) \operatorname{csgn}\left(\frac{I d e^{2 b x+2 a}}{e^{2 b x+2 a}+1}\right) x^4}{16}$$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int \operatorname{arccoth}(c + d \coth(b x + a))\, dx$$

Optimal(type 4, 138 leaves, 7 steps):

$$x\,\text{arccoth}(c+d\coth(bx+a))+\frac{x\ln\left(1-\frac{(1-c-d)\,e^{2bx+2a}}{1-c+d}\right)}{2}-\frac{x\ln\left(1-\frac{(1+c+d)\,e^{2bx+2a}}{1+c-d}\right)}{2}+\frac{\text{polylog}\left(2,\frac{(1-c-d)\,e^{2bx+2a}}{1-c+d}\right)}{4b}-\frac{\text{polylog}\left(2,\frac{(1+c+d)\,e^{2bx+2a}}{1+c-d}\right)}{4b}$$

Result(type 4, 305 leaves):

$$\frac{\text{arccoth}(c+d\coth(bx+a))\ln(d\coth(bx+a)+d)}{2b}-\frac{\text{arccoth}(c+d\coth(bx+a))\ln(d\coth(bx+a)-d)}{2b}-\frac{\text{dilog}\left(\frac{d\coth(bx+a)+c-1}{c+d-1}\right)}{4b}-\frac{\ln(d\coth(bx+a)-d)\ln\left(\frac{d\coth(bx+a)+c-1}{c+d-1}\right)}{4b}+\frac{\text{dilog}\left(\frac{d\coth(bx+a)+c+1}{1+c+d}\right)}{4b}+\frac{\ln(d\coth(bx+a)-d)\ln\left(\frac{d\coth(bx+a)+c+1}{1+c+d}\right)}{4b}-\frac{\text{dilog}\left(\frac{d\coth(bx+a)+c+1}{1+c-d}\right)}{4b}-\frac{\ln(d\coth(bx+a)+d)\ln\left(\frac{d\coth(bx+a)+c+1}{1+c-d}\right)}{4b}+\frac{\text{dilog}\left(\frac{d\coth(bx+a)+c-1}{c-d-1}\right)}{4b}+\frac{\ln(d\coth(bx+a)+d)\ln\left(\frac{d\coth(bx+a)+c-1}{c-d-1}\right)}{4b}$$

Problem 59: Result more than twice size of optimal antiderivative.

$$\int \text{arccoth}(1+d+d\coth(bx+a))\,dx$$

Optimal(type 4, 61 leaves, 5 steps):

$$\frac{bx^2}{2}+x\,\text{arccoth}(1+d+d\coth(bx+a))-\frac{x\ln\left(1-(1+d)\,e^{2bx+2a}\right)}{2}-\frac{\text{polylog}\left(2,(1+d)\,e^{2bx+2a}\right)}{4b}$$

Result(type 4, 246 leaves):

$$\frac{\text{arccoth}(1+d+d\coth(bx+a))\ln(d\coth(bx+a)+d)}{2b}-\frac{\text{arccoth}(1+d+d\coth(bx+a))\ln(d\coth(bx+a)-d)}{2b}-\frac{\text{dilog}\left(\frac{d\coth(bx+a)+d}{2d}\right)}{4b}-\frac{\ln(d\coth(bx+a)-d)\ln\left(\frac{d\coth(bx+a)+d}{2d}\right)}{4b}+\frac{\text{dilog}\left(\frac{d\coth(bx+a)+d+2}{2d+2}\right)}{4b}+\frac{\ln(d\coth(bx+a)-d)\ln\left(\frac{d\coth(bx+a)+d+2}{2d+2}\right)}{4b}+\frac{\ln(d\coth(bx+a)+d)^2}{8b}-\frac{\text{dilog}\left(\frac{d\coth(bx+a)}{2}+\frac{d}{2}+1\right)}{4b}-\frac{\ln(d\coth(bx+a)+d)\ln\left(\frac{d\coth(bx+a)}{2}+\frac{d}{2}+1\right)}{4b}$$

Problem 60: Result more than twice size of optimal antiderivative.

$$\int x^2 \operatorname{arccoth}(1-d-d\coth(bx+a))\,dx$$

Optimal(type 4, 121 leaves, 7 steps):

$$\frac{b x^4}{12}+\frac{x^3 \operatorname{arccoth}(1-d-d\coth(bx+a))}{3}-\frac{x^3 \ln\left(1-(1-d)\,e^{2bx+2a}\right)}{6}-\frac{x^2 \operatorname{polylog}\left(2,(1-d)\,e^{2bx+2a}\right)}{4b}+\frac{x \operatorname{polylog}\left(3,(1-d)\,e^{2bx+2a}\right)}{4b^2}-\frac{\operatorname{polylog}\left(4,(1-d)\,e^{2bx+2a}\right)}{8b^3}$$

Result(type 4, 1778 leaves):

$$\frac{b x^4}{12}+\frac{\operatorname{polylog}\left(2,(1-d)\,e^{2bx+2a}\right)x^2}{4b(d-1)}-\frac{\operatorname{polylog}\left(2,(1-d)\,e^{2bx+2a}\right)a^2}{4b^3(d-1)}-\frac{\operatorname{polylog}\left(3,(1-d)\,e^{2bx+2a}\right)x}{4b^2(d-1)}-\frac{a^3\ln\left(e^{2bx+2a}d-e^{2bx+2a}+1\right)}{6b^3(d-1)}$$
$$-\frac{d\operatorname{polylog}\left(4,(1-d)\,e^{2bx+2a}\right)}{8b^3(d-1)}-\frac{\ln\left(1+(d-1)\,e^{2bx+2a}\right)x a^2}{2b^2(d-1)}+\frac{a^2\ln\left(1+e^{bx+a}\sqrt{1-d}\right)x}{2b^2(d-1)}+\frac{a^2\ln\left(1-e^{bx+a}\sqrt{1-d}\right)x}{2b^2(d-1)}$$
$$-\frac{d a^3\ln\left(1+e^{bx+a}\sqrt{1-d}\right)}{2b^3(d-1)}-\frac{d a^3\ln\left(1-e^{bx+a}\sqrt{1-d}\right)}{2b^3(d-1)}-\frac{d a^2\operatorname{dilog}\left(1+e^{bx+a}\sqrt{1-d}\right)}{2b^3(d-1)}-\frac{d a^2\operatorname{dilog}\left(1-e^{bx+a}\sqrt{1-d}\right)}{2b^3(d-1)}$$
$$+\frac{\operatorname{polylog}\left(4,(1-d)\,e^{2bx+2a}\right)}{8b^3(d-1)}+\frac{I\pi x^3\operatorname{csgn}\left(I e^{2bx+2a}\right)^3}{12}-\frac{I\pi\operatorname{csgn}\left(\frac{I\left(e^{2bx+2a}d-e^{2bx+2a}+1\right)}{e^{2bx+2a}-1}\right)^2 x^3}{6}-\frac{d\ln\left(1+(d-1)\,e^{2bx+2a}\right)x^3}{6(d-1)}$$
$$-\frac{\ln\left(1+(d-1)\,e^{2bx+2a}\right)a^3}{3b^3(d-1)}+\frac{a^3\ln\left(1+e^{bx+a}\sqrt{1-d}\right)}{2b^3(d-1)}+\frac{a^3\ln\left(1-e^{bx+a}\sqrt{1-d}\right)}{2b^3(d-1)}+\frac{a^2\operatorname{dilog}\left(1+e^{bx+a}\sqrt{1-d}\right)}{2b^3(d-1)}$$
$$+\frac{a^2\operatorname{dilog}\left(1-e^{bx+a}\sqrt{1-d}\right)}{2b^3(d-1)}+\frac{d a^3\ln\left(e^{2bx+2a}d-e^{2bx+2a}+1\right)}{6b^3(d-1)}-\frac{I\pi\operatorname{csgn}\left(\frac{I d e^{2bx+2a}}{e^{2bx+2a}-1}\right)^3 x^3}{12}+\frac{I\pi x^3\operatorname{csgn}\left(I e^{bx+a}\right)^2\operatorname{csgn}\left(I e^{2bx+2a}\right)}{12}$$
$$-\frac{I\pi x^3\operatorname{csgn}\left(I e^{bx+a}\right)\operatorname{csgn}\left(I e^{2bx+2a}\right)^2}{6}+\frac{I\pi\operatorname{csgn}\left(I\left(e^{2bx+2a}d-e^{2bx+2a}+1\right)\right)\operatorname{csgn}\left(\frac{I\left(e^{2bx+2a}d-e^{2bx+2a}+1\right)}{e^{2bx+2a}-1}\right)^2 x^3}{12}$$
$$-\frac{I\pi\operatorname{csgn}\left(I e^{2bx+2a}\right)\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}-1}\right)^2 x^3}{12}-\frac{I\pi\operatorname{csgn}(I d)\operatorname{csgn}\left(\frac{I d e^{2bx+2a}}{e^{2bx+2a}-1}\right)^2 x^3}{12}$$
$$+\frac{I\pi\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}-1}\right)\operatorname{csgn}\left(\frac{I\left(e^{2bx+2a}d-e^{2bx+2a}+1\right)}{e^{2bx+2a}-1}\right)^2 x^3}{12}-\frac{I\pi\operatorname{csgn}\left(\frac{I}{e^{2bx+2a}-1}\right)\operatorname{csgn}\left(\frac{I e^{2bx+2a}}{e^{2bx+2a}-1}\right)^2 x^3}{12}$$
$$+\frac{x^3\ln\left(e^{2bx+2a}d-e^{2bx+2a}+1\right)}{6}-\frac{d\operatorname{polylog}\left(2,(1-d)\,e^{2bx+2a}\right)x^2}{4b(d-1)}+\frac{d\operatorname{polylog}\left(2,(1-d)\,e^{2bx+2a}\right)a^2}{4b^3(d-1)}+\frac{d\operatorname{polylog}\left(3,(1-d)\,e^{2bx+2a}\right)x}{4b^2(d-1)}$$

$$+\frac{I\pi\,\mathrm{csgn}\left(\frac{I\,d\,e^{2bx+2a}}{e^{2bx+2a}-1}\right)^{2}x^{3}}{6}+\frac{I\pi\,\mathrm{csgn}\left(\frac{I\left(e^{2bx+2a}d-e^{2bx+2a}+1\right)}{e^{2bx+2a}-1}\right)^{3}x^{3}}{12}+\frac{d\ln\left(1+(d-1)\,e^{2bx+2a}\right)a^{3}}{3\,b^{3}\,(d-1)}-\frac{x^{3}\ln\left(e^{bx+a}\right)}{3}-\frac{\ln(d)\,x^{3}}{6}$$

$$+\frac{\ln\left(1+(d-1)\,e^{2bx+2a}\right)x^{3}}{6\,(d-1)}+\frac{I\pi\,\mathrm{csgn}\left(\frac{I\,e^{2bx+2a}}{e^{2bx+2a}-1}\right)^{3}x^{3}}{12}-\frac{I\pi\,\mathrm{csgn}\left(\frac{I\,e^{2bx+2a}}{e^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{I\,d\,e^{2bx+2a}}{e^{2bx+2a}-1}\right)^{2}x^{3}}{12}$$

$$+\frac{d\ln\left(1+(d-1)\,e^{2bx+2a}\right)x\,a^{2}}{2\,b^{2}\,(d-1)}-\frac{d\,a^{2}\ln\left(1+e^{bx+a}\sqrt{1-d}\right)x}{2\,b^{2}\,(d-1)}-\frac{d\,a^{2}\ln\left(1-e^{bx+a}\sqrt{1-d}\right)x}{2\,b^{2}\,(d-1)}$$

$$+\frac{I\pi\,\mathrm{csgn}\left(I\,e^{2bx+2a}\right)\mathrm{csgn}\left(\frac{I}{e^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{I\,e^{2bx+2a}}{e^{2bx+2a}-1}\right)x^{3}}{12}+\frac{I\pi\,\mathrm{csgn}(I\,d)\,\mathrm{csgn}\left(\frac{I\,e^{2bx+2a}}{e^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{I\,d\,e^{2bx+2a}}{e^{2bx+2a}-1}\right)x^{3}}{12}$$

$$-\frac{I\pi\,\mathrm{csgn}\left(\frac{I}{e^{2bx+2a}-1}\right)\mathrm{csgn}\left(I\left(e^{2bx+2a}d-e^{2bx+2a}+1\right)\right)\mathrm{csgn}\left(\frac{I\left(e^{2bx+2a}d-e^{2bx+2a}+1\right)}{e^{2bx+2a}-1}\right)x^{3}}{12}$$

Problem 61: Result more than twice size of optimal antiderivative.

$$\int \mathrm{arccoth}(1-d-d\coth(bx+a))\,dx$$

Optimal(type 4, 68 leaves, 5 steps):

$$\frac{b\,x^{2}}{2}+x\,\mathrm{arccoth}(1-d-d\coth(bx+a))-\frac{x\ln\left(1-(1-d)\,e^{2bx+2a}\right)}{2}-\frac{\mathrm{polylog}\left(2,\,(1-d)\,e^{2bx+2a}\right)}{4\,b}$$

Result(type 4, 270 leaves):

$$\frac{\mathrm{arccoth}(1-d-d\coth(bx+a))\ln(-d-d\coth(bx+a))}{2\,b}-\frac{\mathrm{arccoth}(1-d-d\coth(bx+a))\ln(-d\coth(bx+a)+d)}{2\,b}+\frac{\ln(-d-d\coth(bx+a))^{2}}{8\,b}$$

$$-\frac{\mathrm{dilog}\left(-\frac{d\coth(bx+a)}{2}-\frac{d}{2}+1\right)}{4\,b}-\frac{\ln(-d-d\coth(bx+a))\ln\left(-\frac{d\coth(bx+a)}{2}-\frac{d}{2}+1\right)}{4\,b}+\frac{\mathrm{dilog}\left(\frac{-d\coth(bx+a)-d+2}{2-2\,d}\right)}{4\,b}$$

$$+\frac{\ln(-d\coth(bx+a)+d)\ln\left(\frac{-d\coth(bx+a)-d+2}{2-2\,d}\right)}{4\,b}-\frac{\mathrm{dilog}\left(-\frac{-d-d\coth(bx+a)}{2\,d}\right)}{4\,b}$$

$$-\frac{\ln(-d\coth(bx+a)+d)\ln\left(-\frac{-d-d\coth(bx+a)}{2\,d}\right)}{4\,b}$$

Problem 63: Result more than twice size of optimal antiderivative.

$$\int (fx+e)^{2}\,\mathrm{arccoth}(\tan(bx+a))\,dx$$

Optimal(type 4, 193 leaves, 10 steps):

$$\frac{(fx+e)^3 \operatorname{arccoth}(\tan(bx+a))}{3f} + \frac{I(fx+e)^3 \arctan(e^{2I(bx+a)})}{3f} - \frac{I(fx+e)^2 \operatorname{polylog}(2, -I e^{2I(bx+a)})}{4b} + \frac{I(fx+e)^2 \operatorname{polylog}(2, I e^{2I(bx+a)})}{4b} + \frac{f(fx+e)\operatorname{polylog}(3, -I e^{2I(bx+a)})}{4b^2} - \frac{f(fx+e)\operatorname{polylog}(3, I e^{2I(bx+a)})}{4b^2} + \frac{I f^2 \operatorname{polylog}(4, -I e^{2I(bx+a)})}{8b^3} - \frac{I f^2 \operatorname{polylog}(4, I e^{2I(bx+a)})}{8b^3}$$

Result(type ?, 5542 leaves): Display of huge result suppressed!

Problem 64: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{arccoth}(c+d\tan(bx+a))\, dx$$

Optimal(type 4, 249 leaves, 9 steps):

$$\frac{x^2 \operatorname{arccoth}(c+d\tan(bx+a))}{2} + \frac{x^2 \ln\left(1+\frac{(1-c+Id)e^{2Ia+2Ibx}}{1-c-Id}\right)}{4} - \frac{x^2 \ln\left(1+\frac{(1+c-Id)e^{2Ia+2Ibx}}{1+c+Id}\right)}{4} - \frac{Ix\operatorname{polylog}\left(2, -\frac{(1-c+Id)e^{2Ia+2Ibx}}{1-c-Id}\right)}{4b} + \frac{Ix\operatorname{polylog}\left(2, -\frac{(1+c-Id)e^{2Ia+2Ibx}}{1+c+Id}\right)}{4b} + \frac{\operatorname{polylog}\left(3, -\frac{(1-c+Id)e^{2Ia+2Ibx}}{1-c-Id}\right)}{8b^2} - \frac{\operatorname{polylog}\left(3, -\frac{(1+c-Id)e^{2Ia+2Ibx}}{1+c+Id}\right)}{8b^2}$$

Result(type ?, 6445 leaves): Display of huge result suppressed!

Problem 65: Result more than twice size of optimal antiderivative.

$$\int \operatorname{arccoth}(c+d\tan(bx+a))\, dx$$

Optimal(type 4, 164 leaves, 7 steps):

$$x\operatorname{arccoth}(c+d\tan(bx+a)) + \frac{x \ln\left(1+\frac{(1-c+Id)e^{2Ia+2Ibx}}{1-c-Id}\right)}{2} - \frac{x \ln\left(1+\frac{(1+c-Id)e^{2Ia+2Ibx}}{1+c+Id}\right)}{2} - \frac{I\operatorname{polylog}\left(2, -\frac{(1-c+Id)e^{2Ia+2Ibx}}{1-c-Id}\right)}{4b} + \frac{I\operatorname{polylog}\left(2, -\frac{(1+c-Id)e^{2Ia+2Ibx}}{1+c+Id}\right)}{4b}$$

Result(type 4, 611 leaves):

$$\frac{\arctan(\tan(bx+a))\operatorname{arccoth}(c+d\tan(bx+a))}{b} + \frac{\arctan\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c-1\right)}{2b} - \frac{\arctan\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2b}$$

$$+\frac{I\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c-1\right)\ln\left(\frac{Id-d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)}{Id+c-1}\right)}{4b}$$

$$-\frac{I\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c-1\right)\ln\left(\frac{Id+d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)}{1-c+Id}\right)}{4b}+\frac{I\,\mathrm{dilog}\left(\frac{Id-d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)}{Id+c-1}\right)}{4b}$$

$$-\frac{I\,\mathrm{dilog}\left(\frac{Id+d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)}{1-c+Id}\right)}{4b}-\frac{I\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)\ln\left(\frac{Id-d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)}{1+c+Id}\right)}{4b}$$

$$+\frac{I\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)\ln\left(\frac{Id+d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)}{Id-c-1}\right)}{4b}-\frac{I\,\mathrm{dilog}\left(\frac{Id-d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)}{1+c+Id}\right)}{4b}$$

$$+\frac{I\,\mathrm{dilog}\left(\frac{Id+d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)}{Id-c-1}\right)}{4b}$$

Problem 66: Result more than twice size of optimal antiderivative.

$$\int x\,\mathrm{arccoth}(1-Id+d\tan(bx+a))\,dx$$

Optimal(type 4, 108 leaves, 6 steps):

$$\frac{Ibx^3}{6}+\frac{x^2\,\mathrm{arccoth}(1-Id+d\tan(bx+a))}{2}-\frac{x^2\ln\left(1+(1-Id)\,e^{2Ia+2Ibx}\right)}{4}+\frac{Ix\,\mathrm{polylog}\left(2,-(1-Id)\,e^{2Ia+2Ibx}\right)}{4b}$$

$$-\frac{\mathrm{polylog}\left(3,-(1-Id)\,e^{2Ia+2Ibx}\right)}{8b^2}$$

Result(type ?, 2248 leaves): Display of huge result suppressed!

Problem 67: Result more than twice size of optimal antiderivative.

$$\int \mathrm{arccoth}(1-Id+d\tan(bx+a))\,dx$$

Optimal(type 4, 76 leaves, 5 steps):

$$\frac{Ibx^2}{2}+x\,\mathrm{arccoth}(1-Id+d\tan(bx+a))-\frac{x\ln\left(1+(1-Id)\,e^{2Ia+2Ibx}\right)}{2}+\frac{I\,\mathrm{polylog}\left(2,-(1-Id)\,e^{2Ia+2Ibx}\right)}{4b}$$

Result(type 4, 291 leaves):

$$-\frac{I\,\mathrm{arccoth}(1-Id+d\tan(bx+a))\ln(-Id+d\tan(bx+a))}{2b}+\frac{I\,\mathrm{arccoth}(1-Id+d\tan(bx+a))\ln(Id+d\tan(bx+a))}{2b}-\frac{I\ln(-Id+d\tan(bx+a))^2}{8b}$$

$$+\frac{I\,\mathrm{dilog}\left(1-\frac{I d}{2}+\frac{d\tan(b x+a)}{2}\right)}{4 b}+\frac{I\ln(-I d+d\tan(b x+a))\ln\left(1-\frac{I d}{2}+\frac{d\tan(b x+a)}{2}\right)}{4 b}-\frac{I\,\mathrm{dilog}\left(\frac{2-I d+d\tan(b x+a)}{-2 I d+2}\right)}{4 b}$$

$$-\frac{I\ln(I d+d\tan(b x+a))\ln\left(\frac{2-I d+d\tan(b x+a)}{-2 I d+2}\right)}{4 b}+\frac{I\,\mathrm{dilog}\left(\frac{\frac{I}{2}(-I d+d\tan(b x+a))}{d}\right)}{4 b}$$

$$+\frac{I\ln(I d+d\tan(b x+a))\ln\left(\frac{\frac{I}{2}(-I d+d\tan(b x+a))}{d}\right)}{4 b}$$

Problem 68: Result more than twice size of optimal antiderivative.

$$\int \mathrm{arccoth}(1+I d-d\tan(b x+a))\,dx$$

Optimal(type 4, 77 leaves, 5 steps):

$$\frac{I b x^2}{2}+x\,\mathrm{arccoth}(1+I d-d\tan(b x+a))-\frac{x\ln\left(1+(1+I d)\,e^{2 I a+2 I b x}\right)}{2}+\frac{I\,\mathrm{polylog}\left(2,-(1+I d)\,e^{2 I a+2 I b x}\right)}{4 b}$$

Result(type 4, 296 leaves):

$$-\frac{I\,\mathrm{arccoth}(1+I d-d\tan(b x+a))\ln(I d-d\tan(b x+a))}{2 b}+\frac{I\,\mathrm{arccoth}(1+I d-d\tan(b x+a))\ln(I d+d\tan(b x+a))}{2 b}-\frac{I\ln(I d-d\tan(b x+a))^2}{8 b}$$

$$+\frac{I\,\mathrm{dilog}\left(1+\frac{I d}{2}-\frac{d\tan(b x+a)}{2}\right)}{4 b}+\frac{I\ln(I d-d\tan(b x+a))\ln\left(1+\frac{I d}{2}-\frac{d\tan(b x+a)}{2}\right)}{4 b}-\frac{I\,\mathrm{dilog}\left(\frac{-2-I d+d\tan(b x+a)}{-2 I d-2}\right)}{4 b}$$

$$-\frac{I\ln(I d+d\tan(b x+a))\ln\left(\frac{-2-I d+d\tan(b x+a)}{-2 I d-2}\right)}{4 b}+\frac{I\,\mathrm{dilog}\left(\frac{\frac{I}{2}(-I d+d\tan(b x+a))}{d}\right)}{4 b}$$

$$+\frac{I\ln(I d+d\tan(b x+a))\ln\left(\frac{\frac{I}{2}(-I d+d\tan(b x+a))}{d}\right)}{4 b}$$

Problem 69: Result more than twice size of optimal antiderivative.

$$\int (f x+e)^2\,\mathrm{arccoth}(\cot(b x+a))\,dx$$

Optimal(type 4, 193 leaves, 10 steps):

$$\frac{(f x+e)^3\,\mathrm{arccoth}(\cot(b x+a))}{3 f}+\frac{I\,(f x+e)^3\arctan\left(e^{2 I(b x+a)}\right)}{3 f}-\frac{I\,(f x+e)^2\,\mathrm{polylog}\left(2,-I e^{2 I(b x+a)}\right)}{4 b}+\frac{I\,(f x+e)^2\,\mathrm{polylog}\left(2,I e^{2 I(b x+a)}\right)}{4 b}$$

$$+\frac{f(fx+e)\,\mathrm{polylog}\left(3,-\mathrm{I}\,\mathrm{e}^{2\,\mathrm{I}\,(bx+a)}\right)}{4\,b^2}-\frac{f(fx+e)\,\mathrm{polylog}\left(3,\mathrm{I}\,\mathrm{e}^{2\,\mathrm{I}\,(bx+a)}\right)}{4\,b^2}+\frac{\mathrm{I}\,f^2\,\mathrm{polylog}\left(4,-\mathrm{I}\,\mathrm{e}^{2\,\mathrm{I}\,(bx+a)}\right)}{8\,b^3}-\frac{\mathrm{I}\,f^2\,\mathrm{polylog}\left(4,\mathrm{I}\,\mathrm{e}^{2\,\mathrm{I}\,(bx+a)}\right)}{8\,b^3}$$

Result(type ?, 5542 leaves): Display of huge result suppressed!

Problem 70: Result more than twice size of optimal antiderivative.

$$\int x^2\,\mathrm{arccoth}(1+\mathrm{I}\,d+d\cot(bx+a))\,dx$$

Optimal(type 4, 136 leaves, 7 steps):

$$\frac{\mathrm{I}\,b\,x^4}{12}+\frac{x^3\,\mathrm{arccoth}(1+\mathrm{I}\,d+d\cot(bx+a))}{3}-\frac{x^3\ln\left(1-(1+\mathrm{I}\,d)\,\mathrm{e}^{2\,\mathrm{I}\,a+2\,\mathrm{I}\,b\,x}\right)}{6}+\frac{\mathrm{I}\,x^2\,\mathrm{polylog}\left(2,(1+\mathrm{I}\,d)\,\mathrm{e}^{2\,\mathrm{I}\,a+2\,\mathrm{I}\,b\,x}\right)}{4\,b}$$
$$-\frac{x\,\mathrm{polylog}\left(3,(1+\mathrm{I}\,d)\,\mathrm{e}^{2\,\mathrm{I}\,a+2\,\mathrm{I}\,b\,x}\right)}{4\,b^2}-\frac{\mathrm{I}\,\mathrm{polylog}\left(4,(1+\mathrm{I}\,d)\,\mathrm{e}^{2\,\mathrm{I}\,a+2\,\mathrm{I}\,b\,x}\right)}{8\,b^3}$$

Result(type ?, 2448 leaves): Display of huge result suppressed!

Problem 71: Result more than twice size of optimal antiderivative.

$$\int x\,\mathrm{arccoth}(1+\mathrm{I}\,d+d\cot(bx+a))\,dx$$

Optimal(type 4, 107 leaves, 6 steps):

$$\frac{\mathrm{I}\,b\,x^3}{6}+\frac{x^2\,\mathrm{arccoth}(1+\mathrm{I}\,d+d\cot(bx+a))}{2}-\frac{x^2\ln\left(1-(1+\mathrm{I}\,d)\,\mathrm{e}^{2\,\mathrm{I}\,a+2\,\mathrm{I}\,b\,x}\right)}{4}+\frac{\mathrm{I}\,x\,\mathrm{polylog}\left(2,(1+\mathrm{I}\,d)\,\mathrm{e}^{2\,\mathrm{I}\,a+2\,\mathrm{I}\,b\,x}\right)}{4\,b}-\frac{\mathrm{polylog}\left(3,(1+\mathrm{I}\,d)\,\mathrm{e}^{2\,\mathrm{I}\,a+2\,\mathrm{I}\,b\,x}\right)}{8\,b^2}$$

Result(type ?, 2350 leaves): Display of huge result suppressed!

Problem 72: Result more than twice size of optimal antiderivative.

$$\int \mathrm{arccoth}(1-\mathrm{I}\,d-d\cot(bx+a))\,dx$$

Optimal(type 4, 77 leaves, 5 steps):

$$\frac{\mathrm{I}\,b\,x^2}{2}+x\,\mathrm{arccoth}(1-\mathrm{I}\,d-d\cot(bx+a))-\frac{x\ln\left(1-(1-\mathrm{I}\,d)\,\mathrm{e}^{2\,\mathrm{I}\,a+2\,\mathrm{I}\,b\,x}\right)}{2}+\frac{\mathrm{I}\,\mathrm{polylog}\left(2,(1-\mathrm{I}\,d)\,\mathrm{e}^{2\,\mathrm{I}\,a+2\,\mathrm{I}\,b\,x}\right)}{4\,b}$$

Result(type 4, 303 leaves):

$$-\frac{\mathrm{I}\,\mathrm{arccoth}(1-\mathrm{I}\,d-d\cot(bx+a))\ln(-\mathrm{I}\,d-d\cot(bx+a))}{2\,b}+\frac{\mathrm{I}\,\mathrm{arccoth}(1-\mathrm{I}\,d-d\cot(bx+a))\ln(\mathrm{I}\,d-d\cot(bx+a))}{2\,b}-\frac{\mathrm{I}\ln(-\mathrm{I}\,d-d\cot(bx+a))^2}{8\,b}$$
$$+\frac{\mathrm{I}\,\mathrm{dilog}\left(1-\frac{\mathrm{I}\,d}{2}-\frac{d\cot(bx+a)}{2}\right)}{4\,b}+\frac{\mathrm{I}\ln(-\mathrm{I}\,d-d\cot(bx+a))\ln\left(1-\frac{\mathrm{I}\,d}{2}-\frac{d\cot(bx+a)}{2}\right)}{4\,b}-\frac{\mathrm{I}\,\mathrm{dilog}\left(\frac{2-\mathrm{I}\,d-d\cot(bx+a)}{-2\,\mathrm{I}\,d+2}\right)}{4\,b}$$
$$-\frac{\mathrm{I}\ln(\mathrm{I}\,d-d\cot(bx+a))\ln\left(\frac{2-\mathrm{I}\,d-d\cot(bx+a)}{-2\,\mathrm{I}\,d+2}\right)}{4\,b}+\frac{\mathrm{I}\,\mathrm{dilog}\left(\frac{\frac{\mathrm{I}}{2}(-\mathrm{I}\,d-d\cot(bx+a))}{d}\right)}{4\,b}$$

$$+\frac{I\ln(Id-d\cot(bx+a))\ln\left(\frac{\frac{I}{2}(-Id-d\cot(bx+a))}{d}\right)}{4b}$$

Problem 73: Result more than twice size of optimal antiderivative.

$$\int x^5(a+b\operatorname{arccoth}(cx))\left(d+e\ln\left(-x^2c^2+1\right)\right)dx$$

Optimal(type 3, 265 leaves, 18 steps):

$$\frac{b(6d-11e)x}{36c^5}-\frac{23bex}{45c^5}+\frac{b(6d-5e)x^3}{108c^3}-\frac{8bex^3}{135c^3}+\frac{b(3d-e)x^5}{90c}-\frac{bex^5}{75c}-\frac{ex^2(a+b\operatorname{arccoth}(cx))}{6c^4}-\frac{ex^4(a+b\operatorname{arccoth}(cx))}{12c^2}$$
$$-\frac{ex^6(a+b\operatorname{arccoth}(cx))}{18}-\frac{b(6d-11e)\operatorname{arctanh}(cx)}{36c^6}+\frac{23be\operatorname{arctanh}(cx)}{45c^6}+\frac{bex\ln\left(-x^2c^2+1\right)}{6c^5}+\frac{bex^3\ln\left(-x^2c^2+1\right)}{18c^3}+\frac{bex^5\ln\left(-x^2c^2+1\right)}{30c}$$
$$-\frac{e(a+b\operatorname{arccoth}(cx))\ln\left(-x^2c^2+1\right)}{6c^6}+\frac{x^6(a+b\operatorname{arccoth}(cx))\left(d+e\ln\left(-x^2c^2+1\right)\right)}{6}$$

Result(type ?, 4033 leaves): Display of huge result suppressed!

Problem 74: Maple result simpler than optimal antiderivative, IF it can be verified!

$$\int\frac{(a+b\operatorname{arccoth}(cx))\left(d+e\ln\left(-x^2c^2+1\right)\right)}{x^6}dx$$

Optimal(type 4, 234 leaves, 24 steps):

$$\frac{7bc^3e}{60x^2}+\frac{2c^2e(a+b\operatorname{arccoth}(cx))}{15x^3}+\frac{2c^4e(a+b\operatorname{arccoth}(cx))}{5x}-\frac{c^5e(a+b\operatorname{arccoth}(cx))^2}{5b}-\frac{5bc^5e\ln(x)}{6}+\frac{19bc^5e\ln\left(-x^2c^2+1\right)}{60}$$
$$-\frac{bc\left(d+e\ln\left(-x^2c^2+1\right)\right)}{20x^4}-\frac{bc^3\left(-x^2c^2+1\right)\left(d+e\ln\left(-x^2c^2+1\right)\right)}{10x^2}-\frac{(a+b\operatorname{arccoth}(cx))\left(d+e\ln\left(-x^2c^2+1\right)\right)}{5x^5}$$
$$+\frac{bc^5\left(d+e\ln\left(-x^2c^2+1\right)\right)\ln\left(1-\frac{1}{-x^2c^2+1}\right)}{10}-\frac{bc^5e\operatorname{polylog}\left(2,\frac{1}{-x^2c^2+1}\right)}{10}$$

Result(type 3, 79 leaves):

$$-\frac{ae\ln\left(-x^2c^2+1\right)}{5x^5}+\frac{a\left(3c^5e\ln(-cx+1)x^5-3c^5e\ln(-cx-1)x^5+6c^4ex^4+2ec^2x^2-3d\right)}{15x^5}$$

Problem 76: Unable to integrate problem.

$$\int\frac{(a+b\operatorname{arccoth}(cx))\left(d+e\ln\left(gx^2+f\right)\right)}{x^2}dx$$

Optimal(type 4, 442 leaves, 38 steps):

$$-\frac{(a+b\operatorname{arccoth}(cx))\,(d+e\ln(g x^2+f))}{x}+\frac{b c\ln\left(-\frac{g x^2}{f}\right)(d+e\ln(g x^2+f))}{2}-\frac{b c\ln\left(\frac{g(-x^2c^2+1)}{fc^2+g}\right)(d+e\ln(g x^2+f))}{2}$$
$$-\frac{b c e\operatorname{polylog}\left(2,\frac{c^2(g x^2+f)}{fc^2+g}\right)}{2}+\frac{b c e\operatorname{polylog}\left(2,1+\frac{g x^2}{f}\right)}{2}+\frac{2 a e\arctan\left(\frac{x\sqrt{g}}{\sqrt{f}}\right)\sqrt{g}}{\sqrt{f}}-\frac{b e\arctan\left(\frac{x\sqrt{g}}{\sqrt{f}}\right)\ln\left(1-\frac{1}{cx}\right)\sqrt{g}}{\sqrt{f}}$$
$$+\frac{b e\arctan\left(\frac{x\sqrt{g}}{\sqrt{f}}\right)\ln\left(1+\frac{1}{cx}\right)\sqrt{g}}{\sqrt{f}}+\frac{b e\arctan\left(\frac{x\sqrt{g}}{\sqrt{f}}\right)\ln\left(-\frac{2(-cx+1)\sqrt{f}\sqrt{g}}{(\mathrm{I}c\sqrt{f}-\sqrt{g})(\sqrt{f}-\mathrm{I}x\sqrt{g})}\right)\sqrt{g}}{\sqrt{f}}$$
$$-\frac{b e\arctan\left(\frac{x\sqrt{g}}{\sqrt{f}}\right)\ln\left(\frac{2(cx+1)\sqrt{f}\sqrt{g}}{(\mathrm{I}c\sqrt{f}+\sqrt{g})(\sqrt{f}-\mathrm{I}x\sqrt{g})}\right)\sqrt{g}}{\sqrt{f}}-\frac{\mathrm{I}b e\operatorname{polylog}\left(2,1+\frac{2(-cx+1)\sqrt{f}\sqrt{g}}{(\mathrm{I}c\sqrt{f}-\sqrt{g})(\sqrt{f}-\mathrm{I}x\sqrt{g})}\right)\sqrt{g}}{2\sqrt{f}}$$
$$+\frac{\mathrm{I}b e\operatorname{polylog}\left(2,1-\frac{2(cx+1)\sqrt{f}\sqrt{g}}{(\mathrm{I}c\sqrt{f}+\sqrt{g})(\sqrt{f}-\mathrm{I}x\sqrt{g})}\right)\sqrt{g}}{2\sqrt{f}}$$

Result(type 8, 26 leaves):

$$\int\frac{(a+b\operatorname{arccoth}(cx))\,(d+e\ln(g x^2+f))}{x^2}\,dx$$

Problem 79: Result more than twice size of optimal antiderivative.

$$\int x^{-1+n}\operatorname{arccoth}(a+b x^n)\,dx$$

Optimal(type 3, 45 leaves, 4 steps):

$$\frac{(a+b x^n)\operatorname{arccoth}(a+b x^n)}{b n}+\frac{\ln\left(1-(a+b x^n)^2\right)}{2 b n}$$

Result(type 3, 117 leaves):

$$\frac{x^n\ln(a+b x^n+1)}{2n}-\frac{x^n\ln(a+b x^n-1)}{2n}+\frac{\ln\left(x^n+\frac{1+a}{b}\right)a}{2 b n}-\frac{\ln\left(x^n+\frac{a-1}{b}\right)a}{2 b n}+\frac{\ln\left(x^n+\frac{1+a}{b}\right)}{2 b n}+\frac{\ln\left(x^n+\frac{a-1}{b}\right)}{2 b n}$$

Test results for the 242 problems in "7.4.2 Exponentials of inverse hyperbolic cotangent functions.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\sqrt{\frac{a x - 1}{a x + 1}}}\, dx$$

Optimal(type 3, 94 leaves, 8 steps):

$$\frac{3 \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2 x^2}}\right)}{8 a^4} + \frac{2 x \sqrt{1 - \frac{1}{a^2 x^2}}}{3 a^3} + \frac{3 x^2 \sqrt{1 - \frac{1}{a^2 x^2}}}{8 a^2} + \frac{x^3 \sqrt{1 - \frac{1}{a^2 x^2}}}{3 a} + \frac{x^4 \sqrt{1 - \frac{1}{a^2 x^2}}}{4}$$

Result(type 3, 192 leaves):

$$-\frac{1}{24 \sqrt{\frac{a x - 1}{a x + 1}}\, \sqrt{(a x - 1)(a x + 1)}\, a^4 \sqrt{a^2}} \Bigg((a x - 1) \Bigg(-6 x (a^2 x^2 - 1)^{3/2} a \sqrt{a^2} - 15 x \sqrt{a^2 x^2 - 1}\, a \sqrt{a^2} - 8 ((a x - 1)(a x + 1))^{3/2} \sqrt{a^2}$$
$$+ 15 \ln\left(\frac{a^2 x + \sqrt{a^2 x^2 - 1}\, \sqrt{a^2}}{\sqrt{a^2}}\right) a - 24 \ln\left(\frac{a^2 x + \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)}}{\sqrt{a^2}}\right) a - 24 \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)} \Bigg)\Bigg)$$

Problem 2: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{\frac{a x - 1}{a x + 1}}}\, dx$$

Optimal(type 3, 53 leaves, 6 steps):

$$\frac{\operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2 x^2}}\right)}{2 a^2} + \frac{x \sqrt{1 - \frac{1}{a^2 x^2}}}{a} + \frac{x^2 \sqrt{1 - \frac{1}{a^2 x^2}}}{2}$$

Result(type 3, 151 leaves):

$$-\frac{(a x - 1) \left(-x \sqrt{a^2 x^2 - 1}\, a \sqrt{a^2} + \ln\left(\frac{a^2 x + \sqrt{a^2 x^2 - 1}\, \sqrt{a^2}}{\sqrt{a^2}}\right) a - 2 \ln\left(\frac{a^2 x + \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)}}{\sqrt{a^2}}\right) a - 2 \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)} \right)}{2 \sqrt{\frac{a x - 1}{a x + 1}}\, \sqrt{(a x - 1)(a x + 1)}\, a^2 \sqrt{a^2}}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{\frac{a x - 1}{a x + 1}}\, x^3}\, dx$$

Optimal(type 3, 32 leaves, 3 steps):

$$-\frac{a^2 \operatorname{arccsc}(a x)}{2}+\frac{a\left(2 a+\frac{1}{x}\right)\sqrt{1-\frac{1}{a^2 x^2}}}{2}$$

Result(type 3, 256 leaves):

$$-\frac{1}{2\sqrt{\frac{a x-1}{a x+1}}\,\sqrt{(a x-1)(a x+1)}\,\sqrt{a^2}\,x^2}\left((a x-1)\left(-2 a^3 x^3\sqrt{a^2 x^2-1}\,\sqrt{a^2}+a^2\arctan\left(\frac{1}{\sqrt{a^2 x^2-1}}\right)\sqrt{a^2}\,x^2+2 a^3\ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2\right.\right.$$
$$-2 a^3\ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right)x^2+2 x\left(a^2 x^2-1\right)^{3/2} a\sqrt{a^2}+a^2\sqrt{a^2 x^2-1}\,\sqrt{a^2}\,x^2-2 a^2\sqrt{(a x-1)(a x+1)}\,\sqrt{a^2}\,x^2+\left(a^2 x^2\right.$$
$$\left.\left.\left.-1\right)^{3/2}\sqrt{a^2}\right)\right)$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int\frac{x^2}{\left(\frac{a x-1}{a x+1}\right)^{3/2}}\,dx$$

Optimal(type 3, 100 leaves, 14 steps):

$$\frac{11\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2 x^2}}\right)}{2 a^3}-\frac{4\sqrt{1-\frac{1}{a^2 x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{14 x\sqrt{1-\frac{1}{a^2 x^2}}}{3 a^2}+\frac{3 x^2\sqrt{1-\frac{1}{a^2 x^2}}}{2 a}+\frac{x^3\sqrt{1-\frac{1}{a^2 x^2}}}{3}$$

Result(type 3, 470 leaves):

$$-\frac{1}{6 a^3\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}\,(a x+1)\left(\frac{a x-1}{a x+1}\right)^{3/2}}\left(-9 a^3 x^3\sqrt{a^2 x^2-1}\,\sqrt{a^2}-2\sqrt{a^2}\,((a x-1)(a x+1))^{3/2} x^2 a^2\right.$$
$$+9 a^3\ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2-42 a^3\ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right)x^2+18 a^2\sqrt{a^2 x^2-1}\,\sqrt{a^2}\,x^2+4\sqrt{a^2}\,((a x-1)(a x$$
$$+1))^{3/2} x a-42 a^2\sqrt{(a x-1)(a x+1)}\,\sqrt{a^2}\,x^2-18\ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x a^2+84\ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right)x a^2$$
$$-9 x\sqrt{a^2 x^2-1}\,a\sqrt{a^2}+10((a x-1)(a x+1))^{3/2}\sqrt{a^2}+84\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}\,x a+9\ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a$$

$$- 42 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)\,(a x+1)}}{\sqrt{a^2}}\right) a - 42 \sqrt{a^2}\,\sqrt{(a x-1)\,(a x+1)}\Bigg)$$

```
Problem 9: Result more than twice size of optimal antiderivative.
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$$\int \frac{x}{\left(\frac{a x-1}{a x+1}\right)^{3/2}}\, dx$$

```
Optimal(type 3, 80 leaves, 12 steps):
```

$$\frac{9 \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2 x^2}}\right)}{2 a^2} - \frac{4 \sqrt{1-\frac{1}{a^2 x^2}}}{a \left(a-\frac{1}{x}\right)} + \frac{3 x \sqrt{1-\frac{1}{a^2 x^2}}}{a} + \frac{x^2 \sqrt{1-\frac{1}{a^2 x^2}}}{2}$$

```
Result(type 3, 420 leaves):
```

$$-\frac{1}{2 a^2 \sqrt{a^2}\,\sqrt{(a x-1)\,(a x+1)}\,(a x+1) \left(\frac{a x-1}{a x+1}\right)^{3/2}} \Bigg(-a^3 x^3 \sqrt{a^2 x^2-1}\,\sqrt{a^2} + a^3 \ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right) x^2$$

$$- 10 a^3 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)\,(a x+1)}}{\sqrt{a^2}}\right) x^2 + 2 a^2 \sqrt{a^2 x^2-1}\,\sqrt{a^2}\, x^2 - 10 a^2 \sqrt{(a x-1)\,(a x+1)}\,\sqrt{a^2}\, x^2 - 2 \ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right) x a^2$$

$$+ 20 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)\,(a x+1)}}{\sqrt{a^2}}\right) x a^2 - x \sqrt{a^2 x^2-1}\, a \sqrt{a^2} + 4\left((a x-1)\,(a x+1)\right)^{3/2} \sqrt{a^2} + 20 \sqrt{a^2}\,\sqrt{(a x-1)\,(a x+1)}\, x a$$

$$+ \ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right) a - 10 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)\,(a x+1)}}{\sqrt{a^2}}\right) a - 10 \sqrt{a^2}\,\sqrt{(a x-1)\,(a x+1)}\Bigg)$$

```
Problem 10: Result more than twice size of optimal antiderivative.
```

$$\int \frac{1}{\left(\frac{a x-1}{a x+1}\right)^{3/2} x^3}\, dx$$

```
Optimal(type 3, 79 leaves, 9 steps):
```

$$-\frac{a^5 \left(1-\frac{1}{a^2 x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} - \frac{3 a^3 \left(1-\frac{1}{a^2 x^2}\right)^{3/2}}{2 \left(a-\frac{1}{x}\right)} + \frac{9 a^2 \operatorname{arccsc}(a x)}{2} - \frac{9 a^2 \sqrt{1-\frac{1}{a^2 x^2}}}{2}$$

```
Result(type 3, 640 leaves):
```

$$\frac{1}{2\sqrt{a^2}\,x^2\sqrt{(ax-1)(ax+1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{3/2}}\Bigg(-6\sqrt{a^2}\sqrt{a^2x^2-1}\,x^5a^5+9\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,x^4a^4$$

$$+6\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)x^4a^5-6\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)x^4a^5+6\sqrt{a^2}\,(a^2x^2-1)^{3/2}x^3a^3+21\sqrt{a^2}\sqrt{a^2x^2-1}\,x^4a^4$$

$$-6\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\,x^4a^4-18\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,x^3a^3-12\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)x^3a^4$$

$$+12\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)x^3a^4-11\sqrt{a^2}\,(a^2x^2-1)^{3/2}x^2a^2-24a^3x^3\sqrt{a^2x^2-1}\sqrt{a^2}-4\sqrt{a^2}\,((ax-1)(ax+1))^{3/2}x^2a^2$$

$$+12\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\,x^3a^3+9a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,x^2+6a^3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)x^2$$

$$-6a^3\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)x^2+4x\,(a^2x^2-1)^{3/2}a\sqrt{a^2}+9a^2\sqrt{a^2x^2-1}\sqrt{a^2}\,x^2-6a^2\sqrt{(ax-1)(ax+1)}\sqrt{a^2}\,x^2$$

$$+(a^2x^2-1)^{3/2}\sqrt{a^2}\Bigg)$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Optimal(type 3, 54 leaves, 6 steps):

$$\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2}$$

Result(type 3, 151 leaves):

$$-\frac{1}{2\sqrt{(ax-1)(ax+1)}\,a^2\sqrt{a^2}}\Bigg(\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\Bigg(-x\sqrt{a^2x^2-1}\,a\sqrt{a^2}+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a$$

$$-2\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a+2\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\Bigg)\Bigg)$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{a x - 1}{a x + 1}}}{x^3}\, dx$$

Optimal(type 3, 34 leaves, 3 steps):

$$\frac{a^2 \operatorname{arccsc}(a x)}{2} + \frac{a \left(2 a - \frac{1}{x}\right) \sqrt{1 - \frac{1}{a^2 x^2}}}{2}$$

Result(type 3, 256 leaves):

$$\frac{1}{2 \sqrt{(a x - 1) (a x + 1)} \sqrt{a^2}\, x^2} \left(\sqrt{\frac{a x - 1}{a x + 1}}\, (a x + 1) \left(2 a^3 x^3 \sqrt{a^2 x^2 - 1} \sqrt{a^2} + a^2 \arctan\left(\frac{1}{\sqrt{a^2 x^2 - 1}}\right) \sqrt{a^2}\, x^2 - 2 a^3 \ln\left(\frac{a^2 x + \sqrt{a^2 x^2 - 1} \sqrt{a^2}}{\sqrt{a^2}}\right) x^2 + 2 a^3 \ln\left(\frac{a^2 x + \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)}}{\sqrt{a^2}}\right) x^2 - 2 x (a^2 x^2 - 1)^{3/2} a \sqrt{a^2} + a^2 \sqrt{a^2 x^2 - 1} \sqrt{a^2}\, x^2 - 2 a^2 \sqrt{(a x - 1) (a x + 1)} \sqrt{a^2}\, x^2 + (a^2 x^2 - 1)^{3/2} \sqrt{a^2}\right)\right)$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{a x - 1}{a x + 1}}}{x^5}\, dx$$

Optimal(type 3, 74 leaves, 5 steps):

$$\frac{3 a^4 \operatorname{arccsc}(a x)}{8} + \frac{a^3 \left(16 a - \frac{9}{x}\right) \sqrt{1 - \frac{1}{a^2 x^2}}}{24} - \frac{a \sqrt{1 - \frac{1}{a^2 x^2}}}{4 x^3} + \frac{a^2 \sqrt{1 - \frac{1}{a^2 x^2}}}{3 x^2}$$

Result(type 3, 307 leaves):

$$\frac{1}{24 \sqrt{(a x - 1) (a x + 1)}\, x^4 \sqrt{a^2}} \left(\sqrt{\frac{a x - 1}{a x + 1}}\, (a x + 1) \left(24 \sqrt{a^2} \sqrt{a^2 x^2 - 1}\, x^5 a^5 + 9 \arctan\left(\frac{1}{\sqrt{a^2 x^2 - 1}}\right) \sqrt{a^2}\, x^4 a^4 - 24 \ln\left(\frac{a^2 x + \sqrt{a^2 x^2 - 1} \sqrt{a^2}}{\sqrt{a^2}}\right) x^4 a^5 + 24 \ln\left(\frac{a^2 x + \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)}}{\sqrt{a^2}}\right) x^4 a^5 - 24 \sqrt{a^2}\, (a^2 x^2 - 1)^{3/2} x^3 a^3 + 9 \sqrt{a^2} \sqrt{a^2 x^2 - 1}\, x^4 a^4 - 24 \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)}\, x^4 a^4 + 15 \sqrt{a^2}\, (a^2 x^2 - 1)^{3/2} x^2 a^2 - 8 x (a^2 x^2 - 1)^{3/2} a \sqrt{a^2} + 6 (a^2 x^2 - 1)^{3/2} \sqrt{a^2}\right)\right)$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(\frac{a x-1}{a x+1}\right)^{3/2}}{x^3}\, dx$$

Optimal(type 3, 75 leaves, 9 steps):

$$-\frac{a^5 \left(1-\frac{1}{a^2 x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3}-\frac{3 a^3 \left(1-\frac{1}{a^2 x^2}\right)^{3/2}}{2 \left(a+\frac{1}{x}\right)}-\frac{9 a^2 \operatorname{arccsc}(a x)}{2}-\frac{9 a^2 \sqrt{1-\frac{1}{a^2 x^2}}}{2}$$

Result(type 3, 640 leaves):

$$-\frac{1}{2 \sqrt{a^2}\, x^2 (a x-1) \sqrt{(a x-1)(a x+1)}}\Bigg(\Bigg(6 \sqrt{a^2} \sqrt{a^2 x^2-1}\, x^5 a^5+9 \arctan\left(\frac{1}{\sqrt{a^2 x^2-1}}\right) \sqrt{a^2}\, x^4 a^4-6 \ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1} \sqrt{a^2}}{\sqrt{a^2}}\right) x^4 a^5$$
$$+6 \ln\left(\frac{a^2 x+\sqrt{a^2} \sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x^4 a^5-6 \sqrt{a^2}\, (a^2 x^2-1)^{3/2} x^3 a^3+21 \sqrt{a^2} \sqrt{a^2 x^2-1}\, x^4 a^4-6 \sqrt{a^2} \sqrt{(a x-1)(a x+1)}\, x^4 a^4$$
$$+18 \arctan\left(\frac{1}{\sqrt{a^2 x^2-1}}\right) \sqrt{a^2}\, x^3 a^3-12 \ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1} \sqrt{a^2}}{\sqrt{a^2}}\right) x^3 a^4+12 \ln\left(\frac{a^2 x+\sqrt{a^2} \sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x^3 a^4-11 \sqrt{a^2}\, (a^2 x^2$$
$$-1)^{3/2} x^2 a^2+24 a^3 x^3 \sqrt{a^2 x^2-1} \sqrt{a^2}-4 \sqrt{a^2}\, ((a x-1)(a x+1))^{3/2} x^2 a^2-12 \sqrt{a^2} \sqrt{(a x-1)(a x+1)}\, x^3 a^3$$
$$+9 a^2 \arctan\left(\frac{1}{\sqrt{a^2 x^2-1}}\right) \sqrt{a^2}\, x^2-6 a^3 \ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1} \sqrt{a^2}}{\sqrt{a^2}}\right) x^2+6 a^3 \ln\left(\frac{a^2 x+\sqrt{a^2} \sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x^2-4 x (a^2 x^2-1)^{3/2} a \sqrt{a^2}$$
$$+9 a^2 \sqrt{a^2 x^2-1} \sqrt{a^2}\, x^2-6 a^2 \sqrt{(a x-1)(a x+1)} \sqrt{a^2}\, x^2+(a^2 x^2-1)^{3/2} \sqrt{a^2}\Bigg) \left(\frac{a x-1}{a x+1}\right)^{3/2}\Bigg)$$

Problem 19: Unable to integrate problem.

$$\int \frac{x^4}{\left(\frac{a x-1}{a x+1}\right)^{1/4}}\, dx$$

Optimal(type 3, 211 leaves, 11 steps):

$$\frac{611 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{1/4} x}{1920 a^4}+\frac{269 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{1/4} x^2}{960 a^3}+\frac{11 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{1/4} x^3}{48 a^2}$$

$$+\frac{9\left(1-\frac{1}{a x}\right)^{3/4}\left(1+\frac{1}{a x}\right)^{1/4} x^4}{40 a}+\frac{\left(1-\frac{1}{a x}\right)^{3/4}\left(1+\frac{1}{a x}\right)^{1/4} x^5}{5}+\frac{31 \arctan\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)}{128 a^5}+\frac{31 \operatorname{arctanh}\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)}{128 a^5}$$

Result(type 8, 116 leaves):

$$\frac{\left(384 x^4 a^4+432 a^3 x^3+440 a^2 x^2+538 a x+611\right)(a x-1)}{1920 a^5\left(\frac{a x-1}{a x+1}\right)^{1/4}}+\frac{\left(\int \frac{31}{256 a^4\left((a x-1)(a x+1)^3\right)^{1/4}} d x\right)\left((a x-1)(a x+1)^3\right)^{1/4}}{\left(\frac{a x-1}{a x+1}\right)^{1/4}(a x+1)}$$

Problem 20: Unable to integrate problem.

$$\int \frac{1}{\left(\frac{a x-1}{a x+1}\right)^{1/4} x} d x$$

Optimal(type 3, 242 leaves, 17 steps):

$$2 \arctan\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)+2 \operatorname{arctanh}\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)+\frac{\ln\left(1-\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}\right)\sqrt{2}}{2}$$

$$-\frac{\ln\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}\right)\sqrt{2}}{2}+\arctan\left(-1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right)\sqrt{2}+\arctan\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right)\sqrt{2}$$

Result(type 8, 21 leaves):

$$\int \frac{1}{\left(\frac{a x-1}{a x+1}\right)^{1/4} x} d x$$

Problem 21: Unable to integrate problem.

$$\int \frac{1}{\left(\frac{a x-1}{a x+1}\right)^{1/4} x^2}\, dx$$

Optimal(type 3, 219 leaves, 13 steps):

$$a\left(1-\frac{1}{a x}\right)^{3/4}\left(1+\frac{1}{a x}\right)^{1/4}+\frac{a \arctan\left(-1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right)\sqrt{2}}{2}+\frac{a \arctan\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right)\sqrt{2}}{2}$$
$$+\frac{a \ln\left(1-\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}\right)\sqrt{2}}{4}-\frac{a \ln\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}\right)\sqrt{2}}{4}$$

Result(type 8, 86 leaves):

$$\frac{a x-1}{x\left(\frac{a x-1}{a x+1}\right)^{1/4}}+\frac{\left(\int \frac{a}{2 x\left((a x-1)\,(a x+1)^3\right)^{1/4}}\, dx\right)\left((a x-1)\,(a x+1)^3\right)^{1/4}}{\left(\frac{a x-1}{a x+1}\right)^{1/4}(a x+1)}$$

Problem 22: Unable to integrate problem.

$$\int \frac{1}{\left(\frac{a x-1}{a x+1}\right)^{1/4} x^3}\, dx$$

Optimal(type 3, 258 leaves, 14 steps):

$$\frac{a^2\left(1-\frac{1}{a x}\right)^{3/4}\left(1+\frac{1}{a x}\right)^{1/4}}{4}+\frac{a^2\left(1-\frac{1}{a x}\right)^{3/4}\left(1+\frac{1}{a x}\right)^{5/4}}{2}+\frac{a^2 \arctan\left(-1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right)\sqrt{2}}{8}$$
$$+\frac{a^2 \arctan\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right)\sqrt{2}}{8}+\frac{a^2 \ln\left(1-\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}\right)\sqrt{2}}{16}$$

$$-\frac{a^2 \ln\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}\right)\sqrt{2}}{16}$$

Result(type 8, 95 leaves):

$$\frac{(a x-1)(3 a x+2)}{4 x^2\left(\frac{a x-1}{a x+1}\right)^{1/4}}+\frac{\left(\int \frac{a^2}{8 x\left((a x-1)(a x+1)^3\right)^{1/4}}\, dx\right)\left((a x-1)(a x+1)^3\right)^{1/4}}{\left(\frac{a x-1}{a x+1}\right)^{1/4}(a x+1)}$$

Problem 23: Unable to integrate problem.

$$\int \frac{x^2}{\left(\frac{a x-1}{a x+1}\right)^{3/4}}\, dx$$

Optimal(type 3, 149 leaves, 9 steps):

$$\frac{23\left(1-\frac{1}{a x}\right)^{1/4}\left(1+\frac{1}{a x}\right)^{3/4} x}{24 a^2}+\frac{7\left(1-\frac{1}{a x}\right)^{1/4}\left(1+\frac{1}{a x}\right)^{3/4} x^2}{12 a}+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\left(1+\frac{1}{a x}\right)^{3/4} x^3}{3}-\frac{17 \arctan\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)}{8 a^3}$$
$$+\frac{17 \operatorname{arctanh}\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)}{8 a^3}$$

Result(type 8, 100 leaves):

$$\frac{\left(8 a^2 x^2+14 a x+23\right)(a x-1)}{24 a^3\left(\frac{a x-1}{a x+1}\right)^{3/4}}+\frac{\left(\int \frac{17}{16 a^2\left((a x-1)^3(a x+1)\right)^{1/4}}\, dx\right)\left((a x-1)^3(a x+1)\right)^{1/4}}{\left(\frac{a x-1}{a x+1}\right)^{3/4}(a x+1)}$$

Problem 24: Unable to integrate problem.

$$\int \frac{x}{\left(\frac{a x-1}{a x+1}\right)^{5/4}}\, dx$$

Optimal(type 3, 146 leaves, 8 steps):

$$-\frac{25\left(1+\frac{1}{a x}\right)^{1/4}}{2 a^2\left(1-\frac{1}{a x}\right)^{1/4}}+\frac{5\left(1+\frac{1}{a x}\right)^{5/4} x}{4 a\left(1-\frac{1}{a x}\right)^{1/4}}+\frac{\left(1+\frac{1}{a x}\right)^{9/4} x^2}{2\left(1-\frac{1}{a x}\right)^{1/4}}+\frac{25 \arctan\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)}{4 a^2}+\frac{25 \operatorname{arctanh}\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)}{4 a^2}$$

Result(type 8, 107 leaves):

$$\frac{(2 a x+11)(a x-1)}{4 a^2\left(\frac{a x-1}{a x+1}\right)^{1/4}}+\frac{\left(\int \frac{-25 a x-7}{8 a^2\left(\frac{1}{a}-x\right)\left((a x-1)(a x+1)^3\right)^{1/4}}\, dx\right)\left((a x-1)(a x+1)^3\right)^{1/4}}{\left(\frac{a x-1}{a x+1}\right)^{1/4}(a x+1)}$$

Problem 25: Unable to integrate problem.

$$\int \frac{1}{\left(\frac{a x-1}{a x+1}\right)^{5/4}}\, dx$$

Optimal(type 3, 114 leaves, 7 steps):

$$-\frac{10\left(1+\frac{1}{a x}\right)^{1/4}}{a\left(1-\frac{1}{a x}\right)^{1/4}}+\frac{\left(1+\frac{1}{a x}\right)^{5/4} x}{\left(1-\frac{1}{a x}\right)^{1/4}}+\frac{5 \arctan\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)}{a}+\frac{5 \operatorname{arctanh}\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)}{a}$$

Result(type 8, 100 leaves):

$$\frac{a x-1}{a\left(\frac{a x-1}{a x+1}\right)^{1/4}}+\frac{\left(\int \frac{-5 a x-3}{2 a\left(\frac{1}{a}-x\right)\left((a x-1)(a x+1)^3\right)^{1/4}}\, dx\right)\left((a x-1)(a x+1)^3\right)^{1/4}}{\left(\frac{a x-1}{a x+1}\right)^{1/4}(a x+1)}$$

Problem 26: Unable to integrate problem.

$$\int \frac{\left(\frac{a x-1}{a x+1}\right)^{1/4}}{x}\, dx$$

Optimal(type 3, 244 leaves, 17 steps):

$$-2\arctan\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)+2\,\mathrm{arctanh}\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)+\frac{\ln\left(1-\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}\right)\sqrt{2}}{2}$$
$$-\frac{\ln\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}\right)\sqrt{2}}{2}-\arctan\left(-1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right)\sqrt{2}-\arctan\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right)\sqrt{2}$$

Result(type 8, 21 leaves):

$$\int \frac{\left(\frac{a x-1}{a x+1}\right)^{1/4}}{x}\, dx$$

Problem 27: Unable to integrate problem.

$$\int \frac{\left(\frac{a x-1}{a x+1}\right)^{1/4}}{x^2}\, dx$$

Optimal(type 3, 220 leaves, 13 steps):

$$-a\left(1-\frac{1}{a x}\right)^{1/4}\left(1+\frac{1}{a x}\right)^{3/4}+\frac{a\arctan\left(-1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right)\sqrt{2}}{2}+\frac{a\arctan\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right)\sqrt{2}}{2}$$
$$-\frac{a\ln\left(1-\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}\right)\sqrt{2}}{4}+\frac{a\ln\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}\right)\sqrt{2}}{4}$$

Result(type 8, 87 leaves):

$$-\frac{(a x+1)\left(\frac{a x-1}{a x+1}\right)^{1/4}}{x}+\frac{\left(\int \frac{a}{2 x\left((a x-1)^3 (a x+1)\right)^{1/4}}\, dx\right)\left(\frac{a x-1}{a x+1}\right)^{1/4}\left((a x-1)^3 (a x+1)\right)^{1/4}}{a x-1}$$

Problem 28: Unable to integrate problem.

$$\int \frac{\left(\frac{a x-1}{a x+1}\right)^{3/4}}{x}\, dx$$

Optimal(type 3, 244 leaves, 17 steps):

$$2 \arctan\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)+2 \operatorname{arctanh}\left(\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right)-\frac{\ln\left(1-\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}\right)\sqrt{2}}{2}$$
$$+\frac{\ln\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}\right)\sqrt{2}}{2}-\arctan\left(-1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right)\sqrt{2}-\arctan\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right)\sqrt{2}$$

Result(type 8, 21 leaves):

$$\int \frac{\left(\frac{a x-1}{a x+1}\right)^{3/4}}{x}\, dx$$

Problem 29: Unable to integrate problem.

$$\int \frac{\left(\frac{a x-1}{a x+1}\right)^{3/4}}{x^2}\, dx$$

Optimal(type 3, 220 leaves, 13 steps):

$$-a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{1/4}+\frac{3a\arctan\left(-1+\frac{\left(1-\frac{1}{ax}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{ax}\right)^{1/4}}\right)\sqrt{2}}{2}+\frac{3a\arctan\left(1+\frac{\left(1-\frac{1}{ax}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{ax}\right)^{1/4}}\right)\sqrt{2}}{2}$$
$$+\frac{3a\ln\left(1-\frac{\left(1-\frac{1}{ax}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{ax}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt{2}}{4}-\frac{3a\ln\left(1+\frac{\left(1-\frac{1}{ax}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{ax}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt{2}}{4}$$

Result(type 8, 87 leaves):

$$-\frac{(ax+1)\left(\frac{ax-1}{ax+1}\right)^{3/4}}{x}+\frac{\left(\int\frac{3a}{2x\left((ax-1)(ax+1)^3\right)^{1/4}}\,dx\right)\left(\frac{ax-1}{ax+1}\right)^{3/4}\left((ax-1)(ax+1)^3\right)^{1/4}}{ax-1}$$

Problem 30: Unable to integrate problem.

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{3/4}}{x^3}\,dx$$

Optimal(type 3, 258 leaves, 14 steps):

$$\frac{3a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{1/4}}{4}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\left(1+\frac{1}{ax}\right)^{1/4}}{2}-\frac{9a^2\arctan\left(-1+\frac{\left(1-\frac{1}{ax}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{ax}\right)^{1/4}}\right)\sqrt{2}}{8}$$
$$-\frac{9a^2\arctan\left(1+\frac{\left(1-\frac{1}{ax}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{ax}\right)^{1/4}}\right)\sqrt{2}}{8}-\frac{9a^2\ln\left(1-\frac{\left(1-\frac{1}{ax}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{ax}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt{2}}{16}$$
$$+\frac{9a^2\ln\left(1+\frac{\left(1-\frac{1}{ax}\right)^{1/4}\sqrt{2}}{\left(1+\frac{1}{ax}\right)^{1/4}}+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt{2}}{16}$$

Result(type 8, 95 leaves):

$$\frac{(ax+1)(5ax-2)\left(\frac{ax-1}{ax+1}\right)^{3/4}}{4x^2}+\frac{\left(\int -\frac{9a^2}{8x\left((ax-1)(ax+1)^3\right)^{1/4}}\,dx\right)\left(\frac{ax-1}{ax+1}\right)^{3/4}\left((ax-1)(ax+1)^3\right)^{1/4}}{ax-1}$$

Problem 31: Unable to integrate problem.

$$\int \frac{x^2}{\left(\frac{-1+x}{1+x}\right)^{1/6}}\,dx$$

Optimal(type 3, 223 leaves, 16 steps):

$$\frac{11\left(1+\frac{1}{x}\right)^{1/6}\left(\frac{-1+x}{x}\right)^{5/6}x}{27}+\frac{7\left(1+\frac{1}{x}\right)^{1/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2}{18}+\frac{\left(1+\frac{1}{x}\right)^{1/6}\left(\frac{-1+x}{x}\right)^{5/6}x^3}{3}+\frac{19\,\mathrm{arctanh}\left(\frac{\left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}\right)}{81}$$
$$-\frac{19\ln\left(1+\frac{\left(1+\frac{1}{x}\right)^{1/3}}{\left(\frac{-1+x}{x}\right)^{1/3}}-\frac{\left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}\right)}{324}+\frac{19\ln\left(1+\frac{\left(1+\frac{1}{x}\right)^{1/3}}{\left(\frac{-1+x}{x}\right)^{1/3}}+\frac{\left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}\right)}{324}$$
$$-\frac{19\arctan\left(\frac{\left(1-\frac{2\left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{162}+\frac{19\arctan\left(\frac{\left(1+\frac{2\left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{162}$$

Result(type 8, 70 leaves):

$$\frac{\left(18x^2+21x+22\right)(-1+x)}{54\left(\frac{-1+x}{1+x}\right)^{1/6}}+\frac{\left(\int \frac{19}{162\left((-1+x)(1+x)^5\right)^{1/6}}\,dx\right)\left((-1+x)(1+x)^5\right)^{1/6}}{\left(\frac{-1+x}{1+x}\right)^{1/6}(1+x)}$$

Problem 32: Unable to integrate problem.

$$\int \frac{1}{\left(\frac{-1+x}{1+x}\right)^{1/6}x^2}\,dx$$

Optimal(type 3, 181 leaves, 14 steps):

$$\left(1+\frac{1}{x}\right)^{1/6}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{2\arctan\left(\frac{\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right)}{3}+\frac{\arctan\left(\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}-\sqrt{3}\right)}{3}+\frac{\arctan\left(\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\sqrt{3}\right)}{3}$$
$$+\frac{\ln\left(1+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}-\frac{\left(\frac{-1+x}{x}\right)^{1/6}\sqrt{3}}{\left(1+\frac{1}{x}\right)^{1/6}}\right)\sqrt{3}}{6}-\frac{\ln\left(1+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}+\frac{\left(\frac{-1+x}{x}\right)^{1/6}\sqrt{3}}{\left(1+\frac{1}{x}\right)^{1/6}}\right)\sqrt{3}}{6}$$

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Result(type 8, 65 leaves):
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$$\frac{-1+x}{x\left(\frac{-1+x}{1+x}\right)^{1/6}}+\frac{\left(\int\frac{1}{3x\left((-1+x)(1+x)^5\right)^{1/6}}\,dx\right)\left((-1+x)(1+x)^5\right)^{1/6}}{\left(\frac{-1+x}{1+x}\right)^{1/6}(1+x)}$$

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Problem 33: Unable to integrate problem.
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$$\int\frac{1}{\left(\frac{-1+x}{1+x}\right)^{1/6}x^3}\,dx$$

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Optimal(type 3, 200 leaves, 15 steps):
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$$\frac{\left(1+\frac{1}{x}\right)^{1/6}\left(\frac{-1+x}{x}\right)^{5/6}}{6}+\frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{2}+\frac{\arctan\left(\frac{\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right)}{9}+\frac{\arctan\left(\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}-\sqrt{3}\right)}{18}$$
$$+\frac{\arctan\left(\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\sqrt{3}\right)}{18}+\frac{\ln\left(1+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}-\frac{\left(\frac{-1+x}{x}\right)^{1/6}\sqrt{3}}{\left(1+\frac{1}{x}\right)^{1/6}}\right)\sqrt{3}}{36}$$
$$-\frac{\ln\left(1+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}+\frac{\left(\frac{-1+x}{x}\right)^{1/6}\sqrt{3}}{\left(1+\frac{1}{x}\right)^{1/6}}\right)\sqrt{3}}{36}$$

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Result(type 8, 71 leaves):
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$$\frac{(-1+x)\,(3+4x)}{6x^2\left(\frac{-1+x}{1+x}\right)^{1/6}}+\frac{\left(\int \frac{1}{18x\left((-1+x)\,(1+x)^5\right)^{1/6}}\,dx\right)\left((-1+x)\,(1+x)^5\right)^{1/6}}{\left(\frac{-1+x}{1+x}\right)^{1/6}(1+x)}$$

Problem 34: Unable to integrate problem.

$$\int \frac{1}{\left(\frac{-1+x}{1+x}\right)^{1/6} x^4}\,dx$$

Optimal(type 3, 221 leaves, 16 steps):

$$\frac{19\left(1+\frac{1}{x}\right)^{1/6}\left(\frac{-1+x}{x}\right)^{5/6}}{54}+\frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{18}+\frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x}+\frac{19\arctan\left(\frac{\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right)}{81}$$
$$+\frac{19\arctan\left(\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}-\sqrt{3}\right)}{162}+\frac{19\arctan\left(\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\sqrt{3}\right)}{162}+\frac{19\ln\left(1+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}-\frac{\left(\frac{-1+x}{x}\right)^{1/6}\sqrt{3}}{\left(1+\frac{1}{x}\right)^{1/6}}\right)\sqrt{3}}{324}$$
$$-\frac{19\ln\left(1+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}+\frac{\left(\frac{-1+x}{x}\right)^{1/6}\sqrt{3}}{\left(1+\frac{1}{x}\right)^{1/6}}\right)\sqrt{3}}{324}$$

Result(type 8, 76 leaves):

$$\frac{(-1+x)\left(22x^2+21x+18\right)}{54x^3\left(\frac{-1+x}{1+x}\right)^{1/6}}+\frac{\left(\int \frac{19}{162x\left((-1+x)\,(1+x)^5\right)^{1/6}}\,dx\right)\left((-1+x)\,(1+x)^5\right)^{1/6}}{\left(\frac{-1+x}{1+x}\right)^{1/6}(1+x)}$$

Problem 35: Unable to integrate problem.

$$\int \frac{x^2}{\left(\frac{-1+x}{1+x}\right)^{1/3}}\,dx$$

Optimal(type 3, 121 leaves, 6 steps):

$$\frac{14\left(1+\frac{1}{x}\right)^{1/3}\left(\frac{-1+x}{x}\right)^{2/3}x}{27}+\frac{4\left(1+\frac{1}{x}\right)^{1/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2}{9}+\frac{\left(1+\frac{1}{x}\right)^{1/3}\left(\frac{-1+x}{x}\right)^{2/3}x^3}{3}-\frac{11\ln\left(\left(1+\frac{1}{x}\right)^{1/3}-\left(\frac{-1+x}{x}\right)^{1/3}\right)}{27}$$

$$-\frac{11\ln(x)}{81}-\frac{22\arctan\left(\frac{\sqrt{3}}{3}+\frac{2\left(\frac{-1+x}{x}\right)^{1/3}\sqrt{3}}{3\left(1+\frac{1}{x}\right)^{1/3}}\right)\sqrt{3}}{81}$$

Result(type 8, 70 leaves):

$$\frac{(9x^2+12x+14)(-1+x)}{27\left(\frac{-1+x}{1+x}\right)^{1/3}}+\frac{\left(\int\frac{22}{81\left((-1+x)(1+x)^2\right)^{1/3}}\,dx\right)\left((-1+x)(1+x)^2\right)^{1/3}}{\left(\frac{-1+x}{1+x}\right)^{1/3}(1+x)}$$

Problem 36: Unable to integrate problem.

$$\int\frac{1}{\left(\frac{-1+x}{1+x}\right)^{1/3}}\,dx$$

Optimal(type 3, 78 leaves, 3 steps):

$$\left(1+\frac{1}{x}\right)^{1/3}\left(\frac{-1+x}{x}\right)^{2/3}x-\ln\left(\left(1+\frac{1}{x}\right)^{1/3}-\left(\frac{-1+x}{x}\right)^{1/3}\right)-\frac{\ln(x)}{3}-\frac{2\arctan\left(\frac{\sqrt{3}}{3}+\frac{2\left(\frac{-1+x}{x}\right)^{1/3}\sqrt{3}}{3\left(1+\frac{1}{x}\right)^{1/3}}\right)\sqrt{3}}{3}$$

Result(type 8, 59 leaves):

$$\frac{-1+x}{\left(\frac{-1+x}{1+x}\right)^{1/3}}+\frac{\left(\int\frac{2}{3\left((-1+x)(1+x)^2\right)^{1/3}}\,dx\right)\left((-1+x)(1+x)^2\right)^{1/3}}{\left(\frac{-1+x}{1+x}\right)^{1/3}(1+x)}$$

Problem 37: Unable to integrate problem.

$$\int\frac{1}{\left(\frac{-1+x}{1+x}\right)^{1/3}x^2}\,dx$$

Optimal(type 3, 81 leaves, 3 steps):

$$\left(1+\frac{1}{x}\right)^{1/3}\left(\frac{-1+x}{x}\right)^{2/3}-\ln\left(1+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right)-\frac{\ln\left(1+\frac{1}{x}\right)}{3}+\frac{2\arctan\left(-\frac{\sqrt{3}}{3}+\frac{2\left(\frac{-1+x}{x}\right)^{1/3}\sqrt{3}}{3\left(1+\frac{1}{x}\right)^{1/3}}\right)\sqrt{3}}{3}$$

Result(type 8, 65 leaves):

$$\frac{-1+x}{x\left(\frac{-1+x}{1+x}\right)^{1/3}}+\frac{\left(\int\frac{2}{3x\left((-1+x)(1+x)^2\right)^{1/3}}\,dx\right)\left((-1+x)(1+x)^2\right)^{1/3}}{\left(\frac{-1+x}{1+x}\right)^{1/3}(1+x)}$$

Problem 38: Unable to integrate problem.

$$\int\frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{1/8}}\,dx$$

Optimal(type 3, 351 leaves, 19 steps):

$$\frac{37\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{1/8}x}{96a^2}+\frac{3\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{1/8}x^2}{8a}+\frac{\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{1/8}x^3}{3}+\frac{11\arctan\left(\frac{\left(1+\frac{1}{ax}\right)^{1/8}}{\left(1-\frac{1}{ax}\right)^{1/8}}\right)}{64a^3}$$

$$+\frac{11\operatorname{arctanh}\left(\frac{\left(1+\frac{1}{ax}\right)^{1/8}}{\left(1-\frac{1}{ax}\right)^{1/8}}\right)}{64a^3}-\frac{11\arctan\left(1-\frac{\left(1+\frac{1}{ax}\right)^{1/8}\sqrt{2}}{\left(1-\frac{1}{ax}\right)^{1/8}}\right)\sqrt{2}}{128a^3}+\frac{11\arctan\left(1+\frac{\left(1+\frac{1}{ax}\right)^{1/8}\sqrt{2}}{\left(1-\frac{1}{ax}\right)^{1/8}}\right)\sqrt{2}}{128a^3}$$

$$-\frac{11\ln\left(1+\frac{\left(1+\frac{1}{ax}\right)^{1/4}}{\left(1-\frac{1}{ax}\right)^{1/4}}-\frac{\left(1+\frac{1}{ax}\right)^{1/8}\sqrt{2}}{\left(1-\frac{1}{ax}\right)^{1/8}}\right)\sqrt{2}}{256a^3}+\frac{11\ln\left(1+\frac{\left(1+\frac{1}{ax}\right)^{1/4}}{\left(1-\frac{1}{ax}\right)^{1/4}}+\frac{\left(1+\frac{1}{ax}\right)^{1/8}\sqrt{2}}{\left(1-\frac{1}{ax}\right)^{1/8}}\right)\sqrt{2}}{256a^3}$$

Result(type 8, 100 leaves):

$$\frac{(32a^2x^2+36ax+37)(ax-1)}{96a^3\left(\frac{ax-1}{ax+1}\right)^{1/8}}+\frac{\left(\int\frac{11}{128a^2\left((ax-1)(ax+1)^7\right)^{1/8}}\,dx\right)\left((ax-1)(ax+1)^7\right)^{1/8}}{\left(\frac{ax-1}{ax+1}\right)^{1/8}(ax+1)}$$

Problem 39: Unable to integrate problem.

$$\int \frac{1}{\left(\frac{a x-1}{a x+1}\right)^{1/8} x^3}\, dx$$

Optimal(type 3, 575 leaves, 26 steps):

$$\frac{a^2 \left(1-\frac{1}{a x}\right)^{7/8} \left(1+\frac{1}{a x}\right)^{1/8}}{8}+\frac{a^2 \left(1-\frac{1}{a x}\right)^{7/8} \left(1+\frac{1}{a x}\right)^{9/8}}{2}-\frac{a^2 \arctan\left(\frac{-\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)\sqrt{2-\sqrt{2}}}{32}$$
$$+\frac{a^2 \arctan\left(\frac{\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)\sqrt{2-\sqrt{2}}}{32}+\frac{a^2 \ln\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}-\frac{\left(1-\frac{1}{a x}\right)^{1/8}\sqrt{2-\sqrt{2}}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right)\sqrt{2-\sqrt{2}}}{64}$$
$$-\frac{a^2 \ln\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\left(1-\frac{1}{a x}\right)^{1/8}\sqrt{2-\sqrt{2}}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right)\sqrt{2-\sqrt{2}}}{64}-\frac{a^2 \arctan\left(\frac{-\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)\sqrt{2+\sqrt{2}}}{32}$$
$$+\frac{a^2 \arctan\left(\frac{\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)\sqrt{2+\sqrt{2}}}{32}+\frac{a^2 \ln\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}-\frac{\left(1-\frac{1}{a x}\right)^{1/8}\sqrt{2+\sqrt{2}}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right)\sqrt{2+\sqrt{2}}}{64}$$
$$-\frac{a^2 \ln\left(1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\left(1-\frac{1}{a x}\right)^{1/8}\sqrt{2+\sqrt{2}}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right)\sqrt{2+\sqrt{2}}}{64}$$

Result(type 8, 95 leaves):

$$\frac{(a x-1)(5 a x+4)}{8 x^{2}\left(\frac{a x-1}{a x+1}\right)^{1/8}}+\frac{\left(\int \frac{a^{2}}{32 x\left((a x-1)(a x+1)^{7}\right)^{1/8}} d x\right)\left((a x-1)(a x+1)^{7}\right)^{1/8}}{\left(\frac{a x-1}{a x+1}\right)^{1/8}(a x+1)}$$

Problem 40: Unable to integrate problem.

$$\int \frac{x^{m}}{\left(\frac{a x-1}{a x+1}\right)^{3/2}} d x$$

Optimal(type 5, 127 leaves, 9 steps):

$$-\frac{3 x^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2},-\frac{1}{2}-\frac{m}{2}\right],\left[\frac{1}{2}-\frac{m}{2}\right], \frac{1}{a^{2} x^{2}}\right)}{1+m}-\frac{x^{m} \operatorname{hypergeom}\left(\left[\frac{1}{2},-\frac{m}{2}\right],\left[1-\frac{m}{2}\right], \frac{1}{a^{2} x^{2}}\right)}{a m}$$
$$+\frac{4 x^{1+m} \operatorname{hypergeom}\left(\left[\frac{3}{2},-\frac{1}{2}-\frac{m}{2}\right],\left[\frac{1}{2}-\frac{m}{2}\right], \frac{1}{a^{2} x^{2}}\right)}{1+m}+\frac{4 x^{m} \operatorname{hypergeom}\left(\left[\frac{3}{2},-\frac{m}{2}\right],\left[1-\frac{m}{2}\right], \frac{1}{a^{2} x^{2}}\right)}{a m}$$

Result(type 8, 21 leaves):

$$\int \frac{x^{m}}{\left(\frac{a x-1}{a x+1}\right)^{3/2}} d x$$

Problem 41: Unable to integrate problem.

$$\int x^{m}\left(\frac{a x-1}{a x+1}\right)^{1/4} d x$$

Optimal(type 6, 37 leaves, 2 steps):

$$\frac{x^{1+m} \mathit{AppellF1}\left(-1-m,-\frac{1}{4}, \frac{1}{4},-m, \frac{1}{a x},-\frac{1}{a x}\right)}{1+m}$$

Result(type 8, 21 leaves):

$$\int x^{m}\left(\frac{a x-1}{a x+1}\right)^{1/4} d x$$

Problem 42: Unable to integrate problem.

$$\int \frac{x^m}{\left(\frac{a x-1}{a x+1}\right)^{1/8}}\, dx$$

Optimal(type 6, 37 leaves, 2 steps):

$$\frac{x^{1+m} AppellF1\left(-1-m, \frac{1}{8}, -\frac{1}{8}, -m, \frac{1}{a x}, -\frac{1}{a x}\right)}{1+m}$$

Result(type 8, 21 leaves):

$$\int \frac{x^m}{\left(\frac{a x-1}{a x+1}\right)^{1/8}}\, dx$$

Problem 43: Unable to integrate problem.

$$\int e^{n \operatorname{arccoth}(a x)} x^m\, dx$$

Optimal(type 6, 41 leaves, 2 steps):

$$\frac{x^{1+m} AppellF1\left(-1-m, \frac{n}{2}, -\frac{n}{2}, -m, \frac{1}{a x}, -\frac{1}{a x}\right)}{1+m}$$

Result(type 8, 13 leaves):

$$\int e^{n \operatorname{arccoth}(a x)} x^m\, dx$$

Problem 44: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{arccoth}(a x)}}{x}\, dx$$

Optimal(type 5, 115 leaves, 4 steps):

$$-\frac{2\left(1+\frac{1}{a x}\right)^{\frac{n}{2}} \operatorname{hypergeom}\left(\left[1, -\frac{n}{2}\right], \left[1-\frac{n}{2}\right], \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n\left(1-\frac{1}{a x}\right)^{\frac{n}{2}}} + \frac{2^{1+\frac{n}{2}} \operatorname{hypergeom}\left(\left[-\frac{n}{2}, -\frac{n}{2}\right], \left[1-\frac{n}{2}\right], \frac{a-\frac{1}{x}}{2 a}\right)}{n\left(1-\frac{1}{a x}\right)^{\frac{n}{2}}}$$

Result(type 8, 13 leaves):

$$\int \frac{e^{n \operatorname{arccoth}(a x)}}{x}\, dx$$

Problem 45: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{arccoth}(a x)}}{x^4} \, dx$$

Optimal(type 5, 143 leaves, 4 steps):

$$\frac{a^3 n \left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{1+\frac{n}{2}}}{6}+\frac{a^2 \left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{1+\frac{n}{2}}}{3 x}$$
$$+\frac{2^{\frac{n}{2}} a^3 \left(n^2+2\right) \left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}} \operatorname{hypergeom}\left(\left[-\frac{n}{2}, 1-\frac{n}{2}\right], \left[2-\frac{n}{2}\right], \frac{a-\frac{1}{x}}{2 a}\right)}{3 (2-n)}$$

Result(type 8, 13 leaves):

$$\int \frac{e^{n \operatorname{arccoth}(a x)}}{x^4} \, dx$$

Problem 46: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{arccoth}(a x)}}{x^5} \, dx$$

Optimal(type 5, 159 leaves, 4 steps):

$$\frac{a^3 \left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{1+\frac{n}{2}} \left(a \left(n^2+6\right)+\frac{2 n}{x}\right)}{24}+\frac{a^2 \left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{1+\frac{n}{2}}}{4 x^2}$$
$$+\frac{2^{-2+\frac{n}{2}} a^4 n \left(n^2+8\right) \left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}} \operatorname{hypergeom}\left(\left[-\frac{n}{2}, 1-\frac{n}{2}\right], \left[2-\frac{n}{2}\right], \frac{a-\frac{1}{x}}{2 a}\right)}{3 (2-n)}$$

Result(type 8, 13 leaves):

$$\int \frac{e^{n \operatorname{arccoth}(a x)}}{x^5} \, dx$$

Problem 51: Unable to integrate problem.

$$\int (-a c x+c)^p \sqrt{\frac{a x-1}{a x+1}} \, dx$$

Optimal(type 5, 88 leaves, 3 steps):

$$\frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p} x\,(-acx+c)^{p}\,\mathrm{hypergeom}\left(\left[-1-p,-\frac{1}{2}-p\right],[-p],\frac{2}{\left(a+\frac{1}{x}\right)x}\right)\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{1+p}$$

Result(type 8, 27 leaves):

$$\int (-acx+c)^{p}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Problem 52: Result more than twice size of optimal antiderivative.

$$\int (-acx+c)\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Optimal(type 3, 55 leaves, 7 steps):

$$-\frac{3c\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}+2cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{acx^2\sqrt{1-\frac{1}{a^2x^2}}}{2}$$

Result(type 3, 152 leaves):

$$\frac{1}{2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a}\left(\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,c\left(-x\sqrt{a^2x^2-1}\,a\sqrt{a^2}+4\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right.\right.$$
$$\left.\left.-4\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a\right)\right)$$

Problem 56: Result more than twice size of optimal antiderivative.

$$\int (-acx+c)^{3}\left(\frac{ax-1}{ax+1}\right)^{3/2}dx$$

Optimal(type 3, 134 leaves, 10 steps):

$$-\frac{315c^3\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+30c^3x\sqrt{1-\frac{1}{a^2x^2}}-\frac{67ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}}{8}+2a^2c^3x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{a^3c^3x^4\sqrt{1-\frac{1}{a^2x^2}}}{4}$$

Result(type 3, 541 leaves):

$$\frac{1}{8a\sqrt{a^2}\,(ax-1)\sqrt{(ax-1)(ax+1)}}\left(\left(-2\sqrt{a^2}\,(a^2x^2-1)^{3/2}x^3a^3+16\sqrt{a^2}\,((ax-1)(ax+1))^{3/2}x^2a^2-4\sqrt{a^2}\,(a^2x^2-1)^{3/2}x^2a^2\right.\right.$$

$$- 69 a^3 x^3 \sqrt{a^2 x^2 - 1}\, \sqrt{a^2} + 32 \sqrt{a^2}\, ((a x - 1)(a x + 1))^{3/2} x a + 384 a^2 \sqrt{(a x - 1)(a x + 1)}\, \sqrt{a^2}\, x^2 - 2 x (a^2 x^2 - 1)^{3/2} a \sqrt{a^2}$$

$$- 138 a^2 \sqrt{a^2 x^2 - 1}\, \sqrt{a^2}\, x^2 + 69 a^3 \ln\left(\frac{a^2 x + \sqrt{a^2 x^2 - 1}\, \sqrt{a^2}}{\sqrt{a^2}}\right) x^2 - 384 a^3 \ln\left(\frac{a^2 x + \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)}}{\sqrt{a^2}}\right) x^2 - 112 ((a x - 1)(a x$$

$$+ 1))^{3/2} \sqrt{a^2} + 768 \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)}\, x a - 69 x \sqrt{a^2 x^2 - 1}\, a \sqrt{a^2} + 138 \ln\left(\frac{a^2 x + \sqrt{a^2 x^2 - 1}\, \sqrt{a^2}}{\sqrt{a^2}}\right) x a^2$$

$$- 768 \ln\left(\frac{a^2 x + \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)}}{\sqrt{a^2}}\right) x a^2 + 384 \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)} + 69 \ln\left(\frac{a^2 x + \sqrt{a^2 x^2 - 1}\, \sqrt{a^2}}{\sqrt{a^2}}\right) a$$

$$- 384 \ln\left(\frac{a^2 x + \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)}}{\sqrt{a^2}}\right) a \Bigg) c^3 \left(\frac{a x - 1}{a x + 1}\right)^{3/2} \Bigg)$$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(\frac{a x - 1}{a x + 1}\right)^{3/2}}{-a c x + c}\, dx$$

Optimal(type 3, 49 leaves, 6 steps):

$$-\frac{\operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2 x^2}}\right)}{a c} + \frac{2 \left(a - \frac{1}{x}\right)}{a^2 c \sqrt{1 - \frac{1}{a^2 x^2}}}$$

Result(type 3, 247 leaves):

$$-\frac{1}{a \sqrt{a^2}\, c (a x - 1) \sqrt{(a x - 1)(a x + 1)}} \Bigg(\Bigg(-a^2 \sqrt{(a x - 1)(a x + 1)}\, \sqrt{a^2}\, x^2 + a^3 \ln\left(\frac{a^2 x + \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)}}{\sqrt{a^2}}\right) x^2 + ((a x - 1)(a x + 1))^{3/}$$

$$^{2} \sqrt{a^2} - 2 \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)}\, x a + 2 \ln\left(\frac{a^2 x + \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)}}{\sqrt{a^2}}\right) x a^2 - \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)}$$

$$+ \ln\left(\frac{a^2 x + \sqrt{a^2}\, \sqrt{(a x - 1)(a x + 1)}}{\sqrt{a^2}}\right) a \Bigg) \left(\frac{a x - 1}{a x + 1}\right)^{3/2} \Bigg)$$

Problem 78: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{\frac{-1+x}{1+x}}\,(1-x)}\,dx$$

Optimal(type 3, 41 leaves, 8 steps):

$$-2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)+\frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}-x\sqrt{1-\frac{1}{x^2}}$$

Result(type 3, 105 leaves):

$$\frac{(x^2-1)^{3/2}-2\sqrt{x^2-1}\,x^2-2\ln\left(x+\sqrt{x^2-1}\right)x^2+4x\sqrt{x^2-1}+4\ln\left(x+\sqrt{x^2-1}\right)x-2\sqrt{x^2-1}-2\ln\left(x+\sqrt{x^2-1}\right)}{(-1+x)\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(-1+x)\,(1+x)}}$$

Problem 79: Unable to integrate problem.

$$\int \frac{x^m\sqrt{-a\,c\,x+c}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Optimal(type 5, 57 leaves, 3 steps):

$$\frac{2\,x^{1+m}\operatorname{hypergeom}\left(\left[-\frac{1}{2},-\frac{3}{2}-m\right],\left[-\frac{1}{2}-m\right],-\frac{1}{a\,x}\right)\sqrt{-a\,c\,x+c}}{(3+2\,m)\sqrt{1-\frac{1}{a\,x}}}$$

Result(type 8, 30 leaves):

$$\int \frac{x^m\sqrt{-a\,c\,x+c}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Problem 89: Unable to integrate problem.

$$\int x^m\sqrt{-a\,c\,x+c}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Optimal(type 5, 117 leaves, 4 steps):

$$-\frac{2\,(5+4m)\,x^{m}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},-\frac{1}{2}-m\right],\left[\frac{1}{2}-m\right],-\frac{1}{a x}\right)\sqrt{-a c x+c}}{a\,(1+2m)\,(3+2m)\,\sqrt{1-\frac{1}{a x}}}+\frac{2\,x^{1+m}\,\sqrt{1+\frac{1}{a x}}\,\sqrt{-a c x+c}}{(3+2m)\,\sqrt{1-\frac{1}{a x}}}$$

Result(type 8, 30 leaves):

$$\int x^{m}\sqrt{-a c x+c}\,\sqrt{\frac{a x-1}{a x+1}}\,dx$$

Problem 98: Unable to integrate problem.

$$\int e^{n\,\mathrm{arccoth}(a x)}\,(-a c x+c)^{-2+\frac{n}{2}}\,dx$$

Optimal(type 5, 80 leaves, 3 steps):

$$-\frac{2\left(1-\frac{1}{a x}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{-1+\frac{n}{2}}x\,(-a c x+c)^{-2+\frac{n}{2}}\,\mathrm{hypergeom}\left(\left[2,1-\frac{n}{2}\right],\left[2-\frac{n}{2}\right],\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

Result(type 8, 23 leaves):

$$\int e^{n\,\mathrm{arccoth}(a x)}\,(-a c x+c)^{-2+\frac{n}{2}}\,dx$$

Problem 99: Unable to integrate problem.

$$\int e^{n\,\mathrm{arccoth}(a x)}\,(-a c x+c)^{p}\,dx$$

Optimal(type 5, 100 leaves, 3 steps):

$$\frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n}{2}-p}\left(1+\frac{1}{a x}\right)^{1+\frac{n}{2}}x\,(-a c x+c)^{p}\,\mathrm{hypergeom}\left(\left[-1-p,\frac{n}{2}-p\right],[-p],\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(1+p)\left(1-\frac{1}{a x}\right)^{\frac{n}{2}}}$$

Result(type 8, 19 leaves):

$$\int e^{n\,\mathrm{arccoth}(a x)}\,(-a c x+c)^{p}\,dx$$

Problem 101: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{arccoth}(a x)}}{\sqrt{-a c x+c}}\, dx$$

Optimal(type 5, 86 leaves, 3 steps):

$$\frac{2 \left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}+\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{1+\frac{n}{2}} x \operatorname{hypergeom}\left(\left[-\frac{1}{2}, \frac{1}{2}+\frac{n}{2}\right], \left[\frac{1}{2}\right], \frac{2}{\left(a+\frac{1}{x}\right) x}\right)}{\left(1-\frac{1}{a x}\right)^{\frac{n}{2}} \sqrt{-a c x+c}}$$

Result(type 8, 19 leaves):

$$\int \frac{e^{n \operatorname{arccoth}(a x)}}{\sqrt{-a c x+c}}\, dx$$

Problem 102: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c-\frac{c}{a x}\right)^3}{\sqrt{\frac{a x-1}{a x+1}}}\, dx$$

Optimal(type 3, 78 leaves, 8 steps):

$$c^3 \left(1-\frac{1}{a^2 x^2}\right)^{3/2} x+\frac{c^3 \operatorname{arccsc}(a x)}{2 a}-\frac{2 c^3 \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2 x^2}}\right)}{a}+\frac{c^3 \left(4 a+\frac{1}{x}\right) \sqrt{1-\frac{1}{a^2 x^2}}}{2 a^2}$$

Result(type 3, 199 leaves):

$$-\frac{1}{2 \sqrt{\frac{a x-1}{a x+1}}\, \sqrt{(a x-1)(a x+1)}\, a^3 x^2 \sqrt{a^2}} \Bigg((a x-1) c^3 \Bigg(-4 a^3 x^3 \sqrt{a^2 x^2-1}\, \sqrt{a^2}+4 x (a^2 x^2-1)^{3/2} a \sqrt{a^2}-a^2 \sqrt{a^2 x^2-1}\, \sqrt{a^2}\, x^2$$
$$+4 a^3 \ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1}\, \sqrt{a^2}}{\sqrt{a^2}}\right) x^2-a^2 \arctan\left(\frac{1}{\sqrt{a^2 x^2-1}}\right) \sqrt{a^2}\, x^2-(a^2 x^2-1)^{3/2} \sqrt{a^2}\Bigg)\Bigg)$$

Problem 103: Result more than twice size of optimal antiderivative.

$$\int \frac{c-\frac{c}{a x}}{\sqrt{\frac{a x-1}{a x+1}}}\, dx$$

Optimal(type 3, 25 leaves, 3 steps):

$$\frac{c \operatorname{arccsc}(a x)}{a}+c x \sqrt{1-\frac{1}{a^2 x^2}}$$

Result(type 3, 62 leaves):

$$\frac{(a x-1)\, c \left(\sqrt{a^2 x^2-1}+\arctan\left(\frac{1}{\sqrt{a^2 x^2-1}}\right)\right)}{\sqrt{\frac{a x-1}{a x+1}}\, \sqrt{(a x-1)\,(a x+1)}\; a}$$

Problem 104: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{\frac{a x-1}{a x+1}}\left(c-\frac{c}{a x}\right)}\, dx$$

Optimal(type 3, 64 leaves, 7 steps):

$$\frac{2 \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2 x^2}}\right)}{a c}-\frac{2\left(a+\frac{1}{x}\right)}{a^2 c \sqrt{1-\frac{1}{a^2 x^2}}}+\frac{x \sqrt{1-\frac{1}{a^2 x^2}}}{c}$$

Result(type 3, 249 leaves):

$$-\frac{1}{a\,(a x-1)\sqrt{a^2}\; c\sqrt{(a x-1)\,(a x+1)}\;\sqrt{\frac{a x-1}{a x+1}}}\left(-2 a^2 \sqrt{(a x-1)\,(a x+1)}\,\sqrt{a^2}\, x^2-2 a^3 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)\,(a x+1)}}{\sqrt{a^2}}\right) x^2+((a x\right.$$

$$-1)\,(a x+1))^{3/2}\sqrt{a^2}+4\sqrt{a^2}\,\sqrt{(a x-1)\,(a x+1)}\; x a+4 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)\,(a x+1)}}{\sqrt{a^2}}\right) x a^2-2\sqrt{a^2}\,\sqrt{(a x-1)\,(a x+1)}$$

$$\left.-2 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)\,(a x+1)}}{\sqrt{a^2}}\right) a\right)$$

Problem 108: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(\frac{a x-1}{a x+1}\right)^{3/2}\left(c-\frac{c}{a x}\right)^{4}}\,dx$$

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Optimal(type 3, 180 leaves, 11 steps):
```

$$\frac{16\left(9 a-\frac{5}{x}\right)}{63 a^{2} c^{4}\left(1-\frac{1}{a^{2} x^{2}}\right)^{7/2}}-\frac{64\left(a+\frac{1}{x}\right)}{9 a^{2} c^{4}\left(1-\frac{1}{a^{2} x^{2}}\right)^{9/2}}-\frac{8\left(21 a+\frac{41}{x}\right)}{105 a^{2} c^{4}\left(1-\frac{1}{a^{2} x^{2}}\right)^{5/2}}+\frac{-735 a-\frac{1417}{x}}{315 a^{2} c^{4}\left(1-\frac{1}{a^{2} x^{2}}\right)^{3/2}}+\frac{7 \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^{2} x^{2}}}\right)}{a c^{4}}$$
$$+\frac{-2205 a-\frac{3149}{x}}{315 a^{2} c^{4}\sqrt{1-\frac{1}{a^{2} x^{2}}}}+\frac{x\sqrt{1-\frac{1}{a^{2} x^{2}}}}{c^{4}}$$

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Result(type 3, 621 leaves):
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$$-\frac{1}{315 a (a x-1)^{4}\sqrt{a^{2}}\, c^{4}\sqrt{(a x-1)(a x+1)}\,(a x+1)\left(\frac{a x-1}{a x+1}\right)^{3/2}}\Bigg(-2205\sqrt{(a x-1)(a x+1)}\sqrt{a^{2}}\, x^{6} a^{6}$$
$$-2205 \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right) x^{6} a^{7}+1890\left((a x-1)(a x+1)\right)^{3/2}\sqrt{a^{2}}\, x^{4} a^{4}+13230\sqrt{(a x-1)(a x+1)}\sqrt{a^{2}}\, x^{5} a^{5}$$
$$+13230 \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right) x^{5} a^{6}-6376\left((a x-1)(a x+1)\right)^{3/2}\sqrt{a^{2}}\, x^{3} a^{3}-33075\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}\, x^{4} a^{4}$$
$$-33075 \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right) x^{4} a^{5}+8646\sqrt{a^{2}}\left((a x-1)(a x+1)\right)^{3/2} x^{2} a^{2}+44100\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}\, x^{3} a^{3}$$
$$+44100 \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right) x^{3} a^{4}-5349\sqrt{a^{2}}\left((a x-1)(a x+1)\right)^{3/2} x a-33075 a^{2}\sqrt{(a x-1)(a x+1)}\sqrt{a^{2}}\, x^{2}$$
$$-33075 a^{3} \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right) x^{2}+1259\left((a x-1)(a x+1)\right)^{3/2}\sqrt{a^{2}}+13230\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}\, x a$$
$$+13230 \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right) x a^{2}-2205\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}-2205 \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right) a\Bigg)$$

```
Problem 110: Result more than twice size of optimal antiderivative.
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$$\int \left(c-\frac{c}{a x}\right)^2 \sqrt{\frac{a x-1}{a x+1}}\, dx$$

Optimal(type 3, 71 leaves, 8 steps):

$$-\frac{3 c^2 \operatorname{arccsc}(a x)}{a}-\frac{3 c^2 \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2 x^2}}\right)}{a}-\frac{c^2 \sqrt{1-\frac{1}{a^2 x^2}}}{a}+c^2 x \sqrt{1-\frac{1}{a^2 x^2}}$$

Result(type 3, 226 leaves):

$$\frac{1}{\sqrt{(a x-1)(a x+1)}\, a^2 x \sqrt{a^2}}\left(\sqrt{\frac{a x-1}{a x+1}}\,(a x+1)\, c^2 \left(-a^2 \sqrt{a^2 x^2-1}\,\sqrt{a^2}\, x^2+4 \sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}\, x a+(a^2 x^2-1)^{3/2} \sqrt{a^2} -3 x \sqrt{a^2 x^2-1}\, a \sqrt{a^2}+\ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right) x a^2-4 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x a^2-3 a \arctan\left(\frac{1}{\sqrt{a^2 x^2-1}}\right) x \sqrt{a^2}\right)\right)$$

Problem 114: Result more than twice size of optimal antiderivative.

$$\int \left(c-\frac{c}{a x}\right)^3 \left(\frac{a x-1}{a x+1}\right)^{3/2} dx$$

Optimal(type 3, 121 leaves, 10 steps):

$$\frac{33 c^3 \operatorname{arccsc}(a x)}{2 a}-\frac{6 c^3 \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2 x^2}}\right)}{a}+\frac{32 c^3 \left(a-\frac{1}{x}\right)}{a^2 \sqrt{1-\frac{1}{a^2 x^2}}}+\frac{6 c^3 \sqrt{1-\frac{1}{a^2 x^2}}}{a}-\frac{c^3 \sqrt{1-\frac{1}{a^2 x^2}}}{2 a^2 x}+c^3 x \sqrt{1-\frac{1}{a^2 x^2}}$$

Result(type 3, 449 leaves):

$$-\frac{1}{2 \sqrt{a^2}\, x^2 a^3 (a x-1) \sqrt{(a x-1)(a x+1)}}\left(\left(-12 \sqrt{a^2}\,\sqrt{a^2 x^2-1}\, x^5 a^5+12 \sqrt{a^2}\,(a^2 x^2-1)^{3/2} x^3 a^3-57 \sqrt{a^2}\,\sqrt{a^2 x^2-1}\, x^4 a^4 +12 \ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right) x^4 a^5-33 \arctan\left(\frac{1}{\sqrt{a^2 x^2-1}}\right) \sqrt{a^2}\, x^4 a^4+32 \sqrt{a^2}\,((a x-1)(a x+1))^{3/2} x^2 a^2+23 \sqrt{a^2}\,(a^2 x^2-1)^{3/2} x^2 a^2 -78 a^3 x^3 \sqrt{a^2 x^2-1}\,\sqrt{a^2}+24 \ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right) x^3 a^4-66 \arctan\left(\frac{1}{\sqrt{a^2 x^2-1}}\right) \sqrt{a^2}\, x^3 a^3+10 x (a^2 x^2-1)^{3/2} a \sqrt{a^2} -33 a^2 \sqrt{a^2 x^2-1}\,\sqrt{a^2}\, x^2+12 a^3 \ln\left(\frac{a^2 x+\sqrt{a^2 x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right) x^2-33 a^2 \arctan\left(\frac{1}{\sqrt{a^2 x^2-1}}\right) \sqrt{a^2}\, x^2-(a^2 x^2-1)^{3/2} \sqrt{a^2}\right) c^3 \left(\frac{a x-1}{a x+1}\right)^{3/2}\right)$$

Problem 115: Result more than twice size of optimal antiderivative.

$$\int \left(c-\frac{c}{a x}\right)\left(\frac{a x-1}{a x+1}\right)^{3/2} dx$$

Optimal(type 3, 69 leaves, 8 steps):

$$\frac{c \operatorname{arccsc}(a x)}{a}-\frac{4 c \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2 x^2}}\right)}{a}+\frac{8 c\left(a-\frac{1}{x}\right)}{a^2 \sqrt{1-\frac{1}{a^2 x^2}}}+c x \sqrt{1-\frac{1}{a^2 x^2}}$$

Result(type 3, 375 leaves):

$$-\frac{1}{a \sqrt{a^2}\,(a x-1) \sqrt{(a x-1)(a x+1)}}\Bigg(\Bigg(-4 a^2 \sqrt{(a x-1)(a x+1)}\, \sqrt{a^2}\, x^2-a^2 \sqrt{a^2 x^2-1}\, \sqrt{a^2}\, x^2+4 a^3 \ln\left(\frac{a^2 x+\sqrt{a^2}\, \sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x^2$$
$$-a^2 \arctan\left(\frac{1}{\sqrt{a^2 x^2-1}}\right) \sqrt{a^2}\, x^2+4\left((a x-1)(a x+1)\right)^{3/2} \sqrt{a^2}-8 \sqrt{a^2}\, \sqrt{(a x-1)(a x+1)}\, x a-2 x \sqrt{a^2 x^2-1}\, a \sqrt{a^2}$$
$$+8 \ln\left(\frac{a^2 x+\sqrt{a^2}\, \sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x a^2-2 a \arctan\left(\frac{1}{\sqrt{a^2 x^2-1}}\right) x \sqrt{a^2}-4 \sqrt{a^2}\, \sqrt{(a x-1)(a x+1)}-\sqrt{a^2 x^2-1}\, \sqrt{a^2}$$
$$+4 \ln\left(\frac{a^2 x+\sqrt{a^2}\, \sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) a-\arctan\left(\frac{1}{\sqrt{a^2 x^2-1}}\right) \sqrt{a^2}\Bigg) c \left(\frac{a x-1}{a x+1}\right)^{3/2}\Bigg)$$

Problem 116: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(\frac{a x-1}{a x+1}\right)^{3/2}}{\left(c-\frac{c}{a x}\right)^2} dx$$

Optimal(type 3, 68 leaves, 6 steps):

$$-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2 x^2}}\right)}{a c^2}-\frac{\left(a-\frac{1}{x}\right) x}{a c^2 \sqrt{1-\frac{1}{a^2 x^2}}}+\frac{2 x \sqrt{1-\frac{1}{a^2 x^2}}}{c^2}$$

Result(type 3, 249 leaves):

$$-\frac{1}{2 a \sqrt{a^2}\, c^2 (a x-1) \sqrt{(a x-1)(a x+1)}}\Bigg(\Bigg(-3 a^2 \sqrt{(a x-1)(a x+1)}\, \sqrt{a^2}\, x^2+2 a^3 \ln\left(\frac{a^2 x+\sqrt{a^2}\, \sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x^2+\left((a x-1)(a x\right.$$
$$\left.+1)\right)^{3/2} \sqrt{a^2}-6 \sqrt{a^2}\, \sqrt{(a x-1)(a x+1)}\, x a+4 \ln\left(\frac{a^2 x+\sqrt{a^2}\, \sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x a^2-3 \sqrt{a^2}\, \sqrt{(a x-1)(a x+1)}$$

$$+2\ln\left(\frac{a^2 x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a\Bigg)\left(\frac{ax-1}{ax+1}\right)^{3/2}\Bigg)$$

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Problem 117: Result more than twice size of optimal antiderivative.
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$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$

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Optimal(type 3, 97 leaves, 7 steps):
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$$\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a c^4}-\frac{ax}{3c^4\left(a-\frac{1}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{8x\sqrt{1-\frac{1}{a^2x^2}}}{3c^4}$$

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Result(type 3, 522 leaves):
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$$-\frac{1}{24a\sqrt{a^2}\,c^4(ax-1)^4\sqrt{(ax-1)(ax+1)}}\Bigg(\Bigg(-45\sqrt{(ax-1)(ax+1)}\sqrt{a^2}\,x^5a^5-24\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)x^5a^6+21((ax$$

$$-1)(ax+1))^{3/2}\sqrt{a^2}\,x^3a^3+45\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\,x^4a^4+24\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)x^4a^5+11\sqrt{a^2}((ax-1)(ax$$

$$+1))^{3/2}x^2a^2+90\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\,x^3a^3+48\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)x^3a^4-5\sqrt{a^2}((ax-1)(ax+1))^{3/2}xa$$

$$-90a^2\sqrt{(ax-1)(ax+1)}\sqrt{a^2}\,x^2-48a^3\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)x^2-19((ax-1)(ax+1))^{3/2}\sqrt{a^2}$$

$$-45\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\,xa-24\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)xa^2+45\sqrt{a^2}\sqrt{(ax-1)(ax+1)}$$

$$+24\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a\Bigg)\left(\frac{ax-1}{ax+1}\right)^{3/2}\Bigg)$$

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Problem 118: Result more than twice size of optimal antiderivative.
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$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^5}\,dx$$

Optimal(type 3, 122 leaves, 9 steps):

$$-\frac{2\left(a+\frac{1}{x}\right)}{5\,a^{2}\,c^{5}\left(1-\frac{1}{a^{2}\,x^{2}}\right)^{5/2}}+\frac{-10\,a-\frac{13}{x}}{15\,a^{2}\,c^{5}\left(1-\frac{1}{a^{2}\,x^{2}}\right)^{3/2}}+\frac{2\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{a^{2}\,x^{2}}}\right)}{a\,c^{5}}+\frac{-30\,a-\frac{41}{x}}{15\,a^{2}\,c^{5}\sqrt{1-\frac{1}{a^{2}\,x^{2}}}}+\frac{x\sqrt{1-\frac{1}{a^{2}\,x^{2}}}}{c^{5}}$$

Result(type 3, 614 leaves):

$$-\frac{1}{30\,a\sqrt{a^{2}}\,c^{5}\sqrt{(a x-1)(a x+1)}\,(a x-1)^{5}}\Bigg(\Bigg(-75\sqrt{(a x-1)(a x+1)}\sqrt{a^{2}}\,x^{6}\,a^{6}-60\ln\left(\frac{a^{2}\,x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right)x^{6}\,a^{7}+45\left((a x-1)(a x+1)\right)^{3/2}\sqrt{a^{2}}\,x^{4}\,a^{4}+150\sqrt{(a x-1)(a x+1)}\sqrt{a^{2}}\,x^{5}\,a^{5}+120\ln\left(\frac{a^{2}\,x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right)x^{5}\,a^{6}+2\left((a x-1)(a x+1)\right)^{3/2}\sqrt{a^{2}}\,x^{3}\,a^{3}+75\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}\,x^{4}\,a^{4}+60\ln\left(\frac{a^{2}\,x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right)x^{4}\,a^{5}-64\sqrt{a^{2}}\left((a x-1)(a x+1)\right)^{3/2}x^{2}\,a^{2}-300\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}\,x^{3}\,a^{3}-240\ln\left(\frac{a^{2}\,x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right)x^{3}\,a^{4}-14\sqrt{a^{2}}\left((a x-1)(a x+1)\right)^{3/2}x\,a+75\,a^{2}\sqrt{(a x-1)(a x+1)}\sqrt{a^{2}}\,x^{2}+60\,a^{3}\ln\left(\frac{a^{2}\,x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right)x^{2}+37\left((a x-1)(a x+1)\right)^{3/2}\sqrt{a^{2}}+150\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}\,x\,a+120\ln\left(\frac{a^{2}\,x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right)x\,a^{2}-75\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}-60\ln\left(\frac{a^{2}\,x+\sqrt{a^{2}}\sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right)a\Bigg)\left(\frac{a x-1}{a x+1}\right)^{3/2}\Bigg)$$

Problem 122: Result more than twice size of optimal antiderivative.

$$\int\frac{a x+1}{(a x-1)\left(c-\frac{c}{a x}\right)^{3/2}}\,dx$$

Optimal(type 3, 81 leaves, 10 steps):

$$-\frac{7}{3\,a\left(c-\frac{c}{a x}\right)^{3/2}}+\frac{x}{\left(c-\frac{c}{a x}\right)^{3/2}}+\frac{7\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}}}{\sqrt{c}}\right)}{a\,c^{3/2}}-\frac{7}{a\,c\sqrt{c-\frac{c}{a x}}}$$

Result(type 3, 264 leaves):

$$\frac{1}{6\sqrt{(ax-1)x}\,c^2 a^{5/2}(ax-1)^3}\Bigg(\sqrt{\frac{c(ax-1)}{ax}}\,x\Bigg(42a^{11/2}\sqrt{(ax-1)x}\,x^3-36a^{9/2}((ax-1)x)^{3/2}x-126a^{9/2}\sqrt{(ax-1)x}\,x^2+28a^{7/2}((ax-1)x)^{3/2}+126a^{7/2}\sqrt{(ax-1)x}\,x+21\ln\left(\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^3a^5-63\ln\left(\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^2a^4-42\sqrt{(ax-1)x}\,a^{5/2}+63\ln\left(\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x a^3-21\ln\left(\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^2\Bigg)\Bigg)$$

Problem 123: Result more than twice size of optimal antiderivative.

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$$

Optimal(type 3, 102 leaves, 11 steps):

$$-\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{9\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}-\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}$$

Result(type 3, 332 leaves):

$$\frac{1}{10\sqrt{(ax-1)x}\,c^3a^{7/2}(ax-1)^4}\Bigg(\sqrt{\frac{c(ax-1)}{ax}}\,x\Bigg(90a^{15/2}\sqrt{(ax-1)x}\,x^4-80a^{13/2}((ax-1)x)^{3/2}x^2-360a^{13/2}\sqrt{(ax-1)x}\,x^3+132a^{11/2}((ax-1)x)^{3/2}x+540a^{11/2}\sqrt{(ax-1)x}\,x^2-60a^{9/2}((ax-1)x)^{3/2}+45\ln\left(\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^4a^7-360a^{9/2}\sqrt{(ax-1)x}\,x-180\ln\left(\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^3a^6+90\sqrt{(ax-1)x}\,a^{7/2}+270\ln\left(\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^2a^5-180\ln\left(\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{2\sqrt{a}}\right)xa^4+45\ln\left(\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^3\Bigg)\Bigg)$$

Problem 129: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}(ax-1)}{ax+1}\,dx$$

Optimal(type 3, 136 leaves, 14 steps):

$$-\frac{5 c^2 \left(c-\frac{c}{a x}\right)^{3/2}}{3 a}+\frac{3 c \left(c-\frac{c}{a x}\right)^{5/2}}{5 a}+\left(c-\frac{c}{a x}\right)^{7/2} x-\frac{11 c^{7/2} \operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}}}{\sqrt{c}}\right)}{a}+\frac{32 c^{7/2} \operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a}$$

$$-\frac{21 c^3 \sqrt{c-\frac{c}{a x}}}{a}$$

Result(type 3, 275 leaves):

$$\frac{1}{30 x^3 a^3 \sqrt{(a x-1) x}\sqrt{\frac{1}{a}}}\left(\sqrt{\frac{c (a x-1)}{a x}}\, c^3 \left(555 a^{5/2} \ln\left(\frac{2\sqrt{x^2 a-x}\sqrt{a}+2 a x-1}{2\sqrt{a}}\right) x^4 \sqrt{\frac{1}{a}}\right.\right.$$

$$-720 a^{5/2} \ln\left(\frac{2\sqrt{(a x-1) x}\sqrt{a}+2 a x-1}{2\sqrt{a}}\right) x^4 \sqrt{\frac{1}{a}}-1110 a^3 \sqrt{x^2 a-x}\, x^4 \sqrt{\frac{1}{a}}+480 a^3 \sqrt{(a x-1) x}\, x^4 \sqrt{\frac{1}{a}}+660 a^2 \left(x^2 a-x\right)^{3/2} x^2 \sqrt{\frac{1}{a}}$$

$$\left.\left.-480 a^2 \sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(a x-1) x}\, a-3 a x+1}{a x+1}\right) x^4-92 a \left(x^2 a-x\right)^{3/2} x \sqrt{\frac{1}{a}}+12 \left(x^2 a-x\right)^{3/2} \sqrt{\frac{1}{a}}\right)\right)$$

Problem 130: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c-\frac{c}{a x}\right)^{5/2} (a x-1)}{a x+1}\, dx$$

Optimal(type 3, 115 leaves, 13 steps):

$$\frac{c \left(c-\frac{c}{a x}\right)^{3/2}}{3 a}+\left(c-\frac{c}{a x}\right)^{5/2} x-\frac{9 c^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}}}{\sqrt{c}}\right)}{a}+\frac{16 c^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a}-\frac{7 c^2 \sqrt{c-\frac{c}{a x}}}{a}$$

Result(type 3, 249 leaves):

$$\frac{1}{6 x^2 a^2 \sqrt{(a x-1) x}\sqrt{\frac{1}{a}}}\left(\sqrt{\frac{c (a x-1)}{a x}}\, c^2 \left(45 a^{3/2} \ln\left(\frac{2\sqrt{x^2 a-x}\sqrt{a}+2 a x-1}{2\sqrt{a}}\right) x^3 \sqrt{\frac{1}{a}}-72 a^{3/2} \ln\left(\frac{2\sqrt{(a x-1) x}\sqrt{a}+2 a x-1}{2\sqrt{a}}\right) x^3 \sqrt{\frac{1}{a}}\right.\right.$$

$$-90 a^2 \sqrt{x^2 a - x}\, x^3 \sqrt{\frac{1}{a}} + 48 a^2 \sqrt{(a x - 1) x}\, x^3 \sqrt{\frac{1}{a}} + 48 a \left(x^2 a - x\right)^{3/2} x \sqrt{\frac{1}{a}} - 48 a \sqrt{2} \ln\left(\frac{2 \sqrt{2} \sqrt{\frac{1}{a}} \sqrt{(a x - 1) x}\, a - 3 a x + 1}{a x + 1}\right) x^3$$

$$-4 \left(x^2 a - x\right)^{3/2} \sqrt{\frac{1}{a}}\Bigg)\Bigg)$$

Problem 131: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c - \frac{c}{a x}}\, (a x - 1)}{a x + 1}\, dx$$

Optimal(type 3, 75 leaves, 11 steps):

$$-\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{a x}}}{\sqrt{c}}\right) \sqrt{c}}{a} + \frac{4 \operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{a x}}\, \sqrt{2}}{2 \sqrt{c}}\right) \sqrt{2}\, \sqrt{c}}{a} + x \sqrt{c - \frac{c}{a x}}$$

Result(type 3, 188 leaves):

$$\frac{1}{2 \sqrt{(a x - 1) x}\, a^{3/2} \sqrt{\frac{1}{a}}} \Bigg(\sqrt{\frac{c (a x - 1)}{a x}}\, x \Bigg(-2 \sqrt{x^2 a - x}\, a^{3/2} \sqrt{\frac{1}{a}} + 4 \sqrt{(a x - 1) x}\, a^{3/2} \sqrt{\frac{1}{a}} + \ln\left(\frac{2 \sqrt{x^2 a - x}\, \sqrt{a} + 2 a x - 1}{2 \sqrt{a}}\right) a \sqrt{\frac{1}{a}}$$

$$-6 \ln\left(\frac{2 \sqrt{(a x - 1) x}\, \sqrt{a} + 2 a x - 1}{2 \sqrt{a}}\right) a \sqrt{\frac{1}{a}} - 4 \sqrt{2} \ln\left(\frac{2 \sqrt{2} \sqrt{\frac{1}{a}} \sqrt{(a x - 1) x}\, a - 3 a x + 1}{a x + 1}\right) \sqrt{a}\Bigg)\Bigg)$$

Problem 139: Unable to integrate problem.

$$\int x^m \sqrt{c - \frac{c}{a x}} \sqrt{\frac{a x - 1}{a x + 1}}\, dx$$

Optimal(type 5, 114 leaves, 4 steps):

$$-\frac{(3 + 4 m)\, x^m \operatorname{hypergeom}\left(\left[\frac{1}{2}, -m\right], [1 - m], -\frac{1}{a x}\right) \sqrt{c - \frac{c}{a x}}}{2 a m (1 + m) \sqrt{1 - \frac{1}{a x}}} + \frac{x^{1 + m} \sqrt{1 + \frac{1}{a x}} \sqrt{c - \frac{c}{a x}}}{(1 + m) \sqrt{1 - \frac{1}{a x}}}$$

Result(type 8, 34 leaves):

$$\int x^m \sqrt{c-\frac{c}{a x}}\,\sqrt{\frac{a x-1}{a x+1}}\,dx$$

Problem 141: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c-\frac{c}{a x}}\,(a x-1)}{(a x+1)\,x}\,dx$$

Optimal(type 3, 69 leaves, 11 steps):

$$2\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}}}{\sqrt{c}}\right)\sqrt{c}-4\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}+2\sqrt{c-\frac{c}{a x}}$$

Result(type 3, 218 leaves):

$$-\frac{1}{x\sqrt{(a x-1)\,x}\,\sqrt{\frac{1}{a}}}\left(\sqrt{\frac{c\,(a x-1)}{a x}}\left(2\sqrt{a}\,\ln\left(\frac{2\sqrt{x^2 a-x}\,\sqrt{a}+2 a x-1}{2\sqrt{a}}\right)x^2\sqrt{\frac{1}{a}}-3\sqrt{a}\,\ln\left(\frac{2\sqrt{(a x-1)\,x}\,\sqrt{a}+2 a x-1}{2\sqrt{a}}\right)x^2\sqrt{\frac{1}{a}}\right.\right.$$

$$\left.\left.-4 a\sqrt{x^2 a-x}\,x^2\sqrt{\frac{1}{a}}+2\sqrt{(a x-1)\,x}\,a x^2\sqrt{\frac{1}{a}}+2\left(x^2 a-x\right)^{3/2}\sqrt{\frac{1}{a}}-2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(a x-1)\,x}\,a-3 a x+1}{a x+1}\right)x^2\right)\right)$$

Problem 142: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c-\frac{c}{a x}}\,(a x-1)}{(a x+1)\,x^3}\,dx$$

Optimal(type 3, 94 leaves, 10 steps):

$$\frac{2 a^2\left(c-\frac{c}{a x}\right)^{3/2}}{3 c}+\frac{2 a^2\left(c-\frac{c}{a x}\right)^{5/2}}{5 c^2}-4 a^2\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}+4 a^2\sqrt{c-\frac{c}{a x}}$$

Result(type 3, 269 leaves):

$$-\frac{1}{15 x^3\sqrt{(a x-1)\,x}\,\sqrt{\frac{1}{a}}}\left(\sqrt{\frac{c\,(a x-1)}{a x}}\left(45 a^{5/2}\ln\left(\frac{2\sqrt{x^2 a-x}\,\sqrt{a}+2 a x-1}{2\sqrt{a}}\right)x^4\sqrt{\frac{1}{a}}-45 a^{5/2}\ln\left(\frac{2\sqrt{(a x-1)\,x}\,\sqrt{a}+2 a x-1}{2\sqrt{a}}\right)x^4\sqrt{\frac{1}{a}}\right.\right.$$

$$-90 a^3 \sqrt{x^2 a - x}\, x^4 \sqrt{\frac{1}{a}} + 30 a^3 \sqrt{(a x - 1) x}\, x^4 \sqrt{\frac{1}{a}} + 60 a^2 (x^2 a - x)^{3/2} x^2 \sqrt{\frac{1}{a}} - 30 a^2 \sqrt{2} \ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(a x - 1) x}\, a - 3 a x + 1}{a x + 1}\right) x^4$$

$$-16 a (x^2 a - x)^{3/2} x \sqrt{\frac{1}{a}} + 6 (x^2 a - x)^{3/2} \sqrt{\frac{1}{a}}\Bigg)\Bigg)$$

Problem 146: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{arccoth}(a x)}}{\sqrt{c - \frac{c}{a x}}}\, dx$$

Optimal(type 6, 93 leaves, 3 steps):

$$-\frac{2^{\frac{1}{2} - \frac{n}{2}} \left(1 + \frac{1}{a x}\right)^{1 + \frac{n}{2}} \mathit{AppellF1}\left(1 + \frac{n}{2}, \frac{1}{2} + \frac{n}{2}, 2, 2 + \frac{n}{2}, \frac{a + \frac{1}{x}}{2 a}, 1 + \frac{1}{a x}\right) \sqrt{1 - \frac{1}{a x}}}{a (2 + n) \sqrt{c - \frac{c}{a x}}}$$

Result(type 8, 23 leaves):

$$\int \frac{e^{n \operatorname{arccoth}(a x)}}{\sqrt{c - \frac{c}{a x}}}\, dx$$

Problem 147: Unable to integrate problem.

$$\int \frac{\left(c - \frac{c}{a x}\right)^p}{\sqrt{\frac{a x - 1}{a x + 1}}}\, dx$$

Optimal(type 6, 76 leaves, 3 steps):

$$-\frac{2^{\frac{1}{2} + p} \left(1 + \frac{1}{a x}\right)^{3/2} \left(c - \frac{c}{a x}\right)^p \mathit{AppellF1}\left(\frac{3}{2}, \frac{1}{2} - p, 2, \frac{5}{2}, \frac{a + \frac{1}{x}}{2 a}, 1 + \frac{1}{a x}\right)}{3 a \left(1 - \frac{1}{a x}\right)^p}$$

Result(type 8, 31 leaves):

$$\int \frac{\left(c-\frac{c}{a x}\right)^{p}}{\sqrt{\frac{a x-1}{a x+1}}}\, dx$$

Problem 169: Result more than twice size of optimal antiderivative.

$$\int \frac{(a x+1)\left(-a^{2} c x^{2}+c\right)^{9/2}}{a x-1}\, dx$$

Optimal(type 3, 144 leaves, 10 steps):

$$-\frac{77 c^{3} x\left(-a^{2} c x^{2}+c\right)^{3/2}}{384}-\frac{77 c^{2} x\left(-a^{2} c x^{2}+c\right)^{5/2}}{480}-\frac{11 c x\left(-a^{2} c x^{2}+c\right)^{7/2}}{80}+\frac{11\left(-a^{2} c x^{2}+c\right)^{9/2}}{90 a}+\frac{(a x+1)\left(-a^{2} c x^{2}+c\right)^{9/2}}{10 a}-\frac{77 c^{9/2} \arctan\left(\frac{a x \sqrt{c}}{\sqrt{-a^{2} c x^{2}+c}}\right)}{256 a}-\frac{77 c^{4} x \sqrt{-a^{2} c x^{2}+c}}{256}$$

Result(type 3, 349 leaves):

$$\frac{x\left(-a^{2} c x^{2}+c\right)^{9/2}}{10}+\frac{9 c x\left(-a^{2} c x^{2}+c\right)^{7/2}}{80}+\frac{21 c^{2} x\left(-a^{2} c x^{2}+c\right)^{5/2}}{160}+\frac{21 c^{3} x\left(-a^{2} c x^{2}+c\right)^{3/2}}{128}+\frac{63 c^{4} x \sqrt{-a^{2} c x^{2}+c}}{256}+\frac{63 c^{5} \arctan\left(\frac{\sqrt{c a^{2}}\, x}{\sqrt{-a^{2} c x^{2}+c}}\right)}{256 \sqrt{c a^{2}}}+\frac{2\left(-\left(x-\frac{1}{a}\right)^{2} a^{2} c-2\left(x-\frac{1}{a}\right) a c\right)^{9/2}}{9 a}-\frac{c\left(-\left(x-\frac{1}{a}\right)^{2} a^{2} c-2\left(x-\frac{1}{a}\right) a c\right)^{7/2} x}{4}-\frac{7 c^{2}\left(-\left(x-\frac{1}{a}\right)^{2} a^{2} c-2\left(x-\frac{1}{a}\right) a c\right)^{5/2} x}{24}-\frac{35 c^{3}\left(-\left(x-\frac{1}{a}\right)^{2} a^{2} c-2\left(x-\frac{1}{a}\right) a c\right)^{3/2} x}{96}-\frac{35 c^{4} \sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2} c-2\left(x-\frac{1}{a}\right) a c}\; x}{64}-\frac{35 c^{5} \arctan\left(\frac{\sqrt{c a^{2}}\, x}{\sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2} c-2\left(x-\frac{1}{a}\right) a c}}\right)}{64 \sqrt{c a^{2}}}$$

Problem 170: Result more than twice size of optimal antiderivative.

$$\int \frac{(a x+1)\left(-a^{2} c x^{2}+c\right)^{7/2}}{a x-1}\, dx$$

Optimal(type 3, 125 leaves, 9 steps):

$$-\frac{15 c^2 x \left(-a^2 c x^2+c\right)^{3/2}}{64}-\frac{3 c x \left(-a^2 c x^2+c\right)^{5/2}}{16}+\frac{9 \left(-a^2 c x^2+c\right)^{7/2}}{56 a}+\frac{(a x+1) \left(-a^2 c x^2+c\right)^{7/2}}{8 a}-\frac{45 c^{7/2} \arctan\left(\frac{a x \sqrt{c}}{\sqrt{-a^2 c x^2+c}}\right)}{128 a}$$
$$-\frac{45 c^3 x \sqrt{-a^2 c x^2+c}}{128}$$

Result(type 3, 295 leaves):

$$\frac{x \left(-a^2 c x^2+c\right)^{7/2}}{8}+\frac{7 c x \left(-a^2 c x^2+c\right)^{5/2}}{48}+\frac{35 c^2 x \left(-a^2 c x^2+c\right)^{3/2}}{192}+\frac{35 c^3 x \sqrt{-a^2 c x^2+c}}{128}+\frac{35 c^4 \arctan\left(\frac{\sqrt{c a^2}\, x}{\sqrt{-a^2 c x^2+c}}\right)}{128 \sqrt{c a^2}}$$
$$+\frac{2 \left(-\left(x-\frac{1}{a}\right)^2 a^2 c-2 \left(x-\frac{1}{a}\right) a c\right)^{7/2}}{7 a}-\frac{c \left(-\left(x-\frac{1}{a}\right)^2 a^2 c-2 \left(x-\frac{1}{a}\right) a c\right)^{5/2} x}{3}-\frac{5 c^2 \left(-\left(x-\frac{1}{a}\right)^2 a^2 c-2 \left(x-\frac{1}{a}\right) a c\right)^{3/2} x}{12}$$
$$-\frac{5 c^3 \sqrt{-\left(x-\frac{1}{a}\right)^2 a^2 c-2 \left(x-\frac{1}{a}\right) a c}\; x}{8}-\frac{5 c^4 \arctan\left(\frac{\sqrt{c a^2}\, x}{\sqrt{-\left(x-\frac{1}{a}\right)^2 a^2 c-2 \left(x-\frac{1}{a}\right) a c}}\right)}{8 \sqrt{c a^2}}$$

Problem 174: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(-a^2 c x^2+c\right)^{5/2} (a x-1)}{a x+1}\, dx$$

Optimal(type 3, 107 leaves, 8 steps):

$$-\frac{7 c x \left(-a^2 c x^2+c\right)^{3/2}}{24}-\frac{7 \left(-a^2 c x^2+c\right)^{5/2}}{30 a}-\frac{(-a x+1) \left(-a^2 c x^2+c\right)^{5/2}}{6 a}-\frac{7 c^{5/2} \arctan\left(\frac{a x \sqrt{c}}{\sqrt{-a^2 c x^2+c}}\right)}{16 a}-\frac{7 c^2 x \sqrt{-a^2 c x^2+c}}{16}$$

Result(type 3, 225 leaves):

$$\frac{x \left(-a^2 c x^2+c\right)^{5/2}}{6}+\frac{5 c x \left(-a^2 c x^2+c\right)^{3/2}}{24}+\frac{5 c^2 x \sqrt{-a^2 c x^2+c}}{16}+\frac{5 c^3 \arctan\left(\frac{\sqrt{c a^2}\, x}{\sqrt{-a^2 c x^2+c}}\right)}{16 \sqrt{c a^2}}-\frac{2 \left(-\left(x+\frac{1}{a}\right)^2 a^2 c+2 \left(x+\frac{1}{a}\right) a c\right)^{5/2}}{5 a}$$
$$-\frac{c \left(-\left(x+\frac{1}{a}\right)^2 a^2 c+2 \left(x+\frac{1}{a}\right) a c\right)^{3/2} x}{2}-\frac{3 c^2 \sqrt{-\left(x+\frac{1}{a}\right)^2 a^2 c+2 \left(x+\frac{1}{a}\right) a c}\; x}{4}-\frac{3 c^3 \arctan\left(\frac{\sqrt{c a^2}\, x}{\sqrt{-\left(x+\frac{1}{a}\right)^2 a^2 c+2 \left(x+\frac{1}{a}\right) a c}}\right)}{4 \sqrt{c a^2}}$$

Problem 184: Result more than twice size of optimal antiderivative.

$$\int \frac{(a x+1)\sqrt{-a^2 c x^2+c}}{(a x-1)\, x^4}\, dx$$

Optimal(type 3, 83 leaves, 8 steps):

$$a^3 \operatorname{arctanh}\left(\frac{\sqrt{-a^2 c x^2+c}}{\sqrt{c}}\right)\sqrt{c}+\frac{\sqrt{-a^2 c x^2+c}}{3 x^3}+\frac{a\sqrt{-a^2 c x^2+c}}{x^2}+\frac{5 a^2\sqrt{-a^2 c x^2+c}}{3 x}$$

Result(type 3, 260 leaves):

$$\frac{(-a^2 c x^2+c)^{3/2}}{3 c x^3}+\frac{a\,(-a^2 c x^2+c)^{3/2}}{c x^2}+\sqrt{c}\,\ln\left(\frac{2 c+2\sqrt{c}\,\sqrt{-a^2 c x^2+c}}{x}\right)a^3-\sqrt{-a^2 c x^2+c}\, a^3+\frac{2 a^2\,(-a^2 c x^2+c)^{3/2}}{c x}+2 a^4 x\sqrt{-a^2 c x^2+c}$$

$$+\frac{2 a^4 c \arctan\left(\frac{\sqrt{c a^2}\, x}{\sqrt{-a^2 c x^2+c}}\right)}{\sqrt{c a^2}}+2 a^3\sqrt{-\left(x-\frac{1}{a}\right)^2 a^2 c-2\left(x-\frac{1}{a}\right) a c}-\frac{2 a^4 c \arctan\left(\frac{\sqrt{c a^2}\, x}{\sqrt{-\left(x-\frac{1}{a}\right)^2 a^2 c-2\left(x-\frac{1}{a}\right) a c}}\right)}{\sqrt{c a^2}}$$

Problem 201: Unable to integrate problem.

$$\int \frac{x^m\sqrt{-a^2 c x^2+c}}{\left(\frac{a x-1}{a x+1}\right)^{3/2}}\, dx$$

Optimal(type 5, 126 leaves, 5 steps):

$$\frac{3 x^m\sqrt{-a^2 c x^2+c}}{a\,(1+m)\sqrt{1-\frac{1}{a^2 x^2}}}+\frac{x^{1+m}\sqrt{-a^2 c x^2+c}}{(2+m)\sqrt{1-\frac{1}{a^2 x^2}}}-\frac{4 x^m \operatorname{hypergeom}([1, 1+m], [2+m], a x)\sqrt{-a^2 c x^2+c}}{a\,(1+m)\sqrt{1-\frac{1}{a^2 x^2}}}$$

Result(type 8, 34 leaves):

$$\int \frac{x^m\sqrt{-a^2 c x^2+c}}{\left(\frac{a x-1}{a x+1}\right)^{3/2}}\, dx$$

Problem 202: Unable to integrate problem.

$$\int e^{n \operatorname{arccoth}(a x)}\,(-a^2 c x^2+c)^2\, dx$$

Optimal(type 5, 75 leaves, 3 steps):

$$\frac{64\,c^2\left(1-\frac{1}{a\,x}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{-3+\frac{n}{2}}\mathrm{hypergeom}\left(\left[6,3-\frac{n}{2}\right],\left[4-\frac{n}{2}\right],\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a\,(6-n)}$$

Result(type 8, 23 leaves):

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(a\,x)}\left(-a^2\,c\,x^2+c\right)^2\,\mathrm{d}x$$

Problem 206: Unable to integrate problem.

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(a\,x)}}{x\left(-a^2\,c\,x^2+c\right)^{3/2}}\,\mathrm{d}x$$

Optimal(type 5, 235 leaves, 5 steps):

$$-\frac{a^3\left(1-\frac{1}{a^2\,x^2}\right)^{3/2}\left(1-\frac{1}{a\,x}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{-\frac{1}{2}+\frac{n}{2}}x^3}{(1+n)\left(-a^2\,c\,x^2+c\right)^{3/2}}+\frac{a^3\left(1-\frac{1}{a^2\,x^2}\right)^{3/2}\left(1-\frac{1}{a\,x}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{-\frac{1}{2}+\frac{n}{2}}x^3}{\left(-n^2+1\right)\left(-a^2\,c\,x^2+c\right)^{3/2}}$$

$$-\frac{2^{\frac{1}{2}+\frac{n}{2}}\,a^3\left(1-\frac{1}{a^2\,x^2}\right)^{3/2}\left(1-\frac{1}{a\,x}\right)^{\frac{1}{2}-\frac{n}{2}}x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2}\right],\left[\frac{3}{2}-\frac{n}{2}\right],\frac{a-\frac{1}{x}}{2\,a}\right)}{(-n+1)\left(-a^2\,c\,x^2+c\right)^{3/2}}$$

Result(type 8, 26 leaves):

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(a\,x)}}{x\left(-a^2\,c\,x^2+c\right)^{3/2}}\,\mathrm{d}x$$

Problem 207: Unable to integrate problem.

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(a\,x)}\,x^4}{\left(-a^2\,c\,x^2+c\right)^{5/2}}\,\mathrm{d}x$$

Optimal(type 5, 411 leaves, 8 steps):

$$-\frac{\left(1-\frac{1}{a^2\,x^2}\right)^{5/2}\left(1-\frac{1}{a\,x}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{-\frac{3}{2}+\frac{n}{2}}x^5}{(3+n)\left(-a^2\,c\,x^2+c\right)^{5/2}}-\frac{(6+n)\left(1-\frac{1}{a^2\,x^2}\right)^{5/2}\left(1-\frac{1}{a\,x}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{-\frac{3}{2}+\frac{n}{2}}x^5}{(1+n)\,(3+n)\left(-a^2\,c\,x^2+c\right)^{5/2}}$$

$$+\frac{\left(n^2+6n+15\right)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{-\frac{3}{2}+\frac{n}{2}}x^5}{(-n+1)(1+n)(3+n)\left(-a^2cx^2+c\right)^{5/2}}$$

$$-\frac{\left(-n^3-2n^2+7n+18\right)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{-\frac{3}{2}+\frac{n}{2}}x^5}{\left(n^4-10n^2+9\right)\left(-a^2cx^2+c\right)^{5/2}}$$

$$-\frac{2\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{-\frac{1}{2}+\frac{n}{2}}x^5\,\mathrm{hypergeom}\left(\left[1,-\frac{1}{2}+\frac{n}{2}\right],\left[\frac{1}{2}+\frac{n}{2}\right],\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{(-n+1)\left(-a^2cx^2+c\right)^{5/2}}$$

Result(type 8, 26 leaves):

$$\int\frac{e^{n\,\mathrm{arccoth}(ax)}x^4}{\left(-a^2cx^2+c\right)^{5/2}}\,dx$$

Problem 211: Unable to integrate problem.

$$\int\frac{\left(-a^2cx^2+c\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Optimal(type 5, 112 leaves, 3 steps):

$$\frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{3}{2}+p}x\left(-a^2cx^2+c\right)^p\,\mathrm{hypergeom}\left(\left[-1-2p,\frac{1}{2}-p\right],[-2p],\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(1+2p)\left(1-\frac{1}{a^2x^2}\right)^p}$$

Result(type 8, 31 leaves):

$$\int\frac{\left(-a^2cx^2+c\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Problem 214: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{\frac{a x-1}{a x+1}}\,\left(c-\frac{c}{a^{2} x^{2}}\right)^{3}}\, dx$$

Optimal(type 3, 216 leaves, 10 steps):

$$-\frac{6}{5 a c^{3} \left(1-\frac{1}{a x}\right)^{5/2} \left(1+\frac{1}{a x}\right)^{3/2}}-\frac{29}{15 a c^{3} \left(1-\frac{1}{a x}\right)^{3/2} \left(1+\frac{1}{a x}\right)^{3/2}}+\frac{x}{c^{3} \left(1-\frac{1}{a x}\right)^{5/2} \left(1+\frac{1}{a x}\right)^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a x}}\,\sqrt{1+\frac{1}{a x}}\right)}{a c^{3}}$$

$$-\frac{34}{5 a c^{3} \left(1+\frac{1}{a x}\right)^{3/2} \sqrt{1-\frac{1}{a x}}}+\frac{21 \sqrt{1-\frac{1}{a x}}}{5 a c^{3} \left(1+\frac{1}{a x}\right)^{3/2}}+\frac{16 \sqrt{1-\frac{1}{a x}}}{5 a c^{3} \sqrt{1+\frac{1}{a x}}}$$

Result(type 3, 713 leaves):

$$-\frac{1}{240 a \left(a x+1\right)^{3} \sqrt{a^{2}}\, \left(a x-1\right)^{3} c^{3} \sqrt{\left(a x-1\right) \left(a x+1\right)}\, \sqrt{\frac{a x-1}{a x+1}}}\Bigg(-525 \sqrt{\left(a x-1\right) \left(a x+1\right)}\, \sqrt{a^{2}}\, x^{7} a^{7}$$

$$-240 \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\, \sqrt{\left(a x-1\right) \left(a x+1\right)}}{\sqrt{a^{2}}}\right) x^{7} a^{8}+285 \left(\left(a x-1\right) \left(a x+1\right)\right)^{3/2} \sqrt{a^{2}}\, x^{5} a^{5}+525 \sqrt{\left(a x-1\right) \left(a x+1\right)}\, \sqrt{a^{2}}\, x^{6} a^{6}$$

$$+240 \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\, \sqrt{\left(a x-1\right) \left(a x+1\right)}}{\sqrt{a^{2}}}\right) x^{6} a^{7}+83 \left(\left(a x-1\right) \left(a x+1\right)\right)^{3/2} \sqrt{a^{2}}\, x^{4} a^{4}+1575 \sqrt{\left(a x-1\right) \left(a x+1\right)}\, \sqrt{a^{2}}\, x^{5} a^{5}$$

$$+720 \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\, \sqrt{\left(a x-1\right) \left(a x+1\right)}}{\sqrt{a^{2}}}\right) x^{5} a^{6}-218 \left(\left(a x-1\right) \left(a x+1\right)\right)^{3/2} \sqrt{a^{2}}\, x^{3} a^{3}-1575 \sqrt{a^{2}}\, \sqrt{\left(a x-1\right) \left(a x+1\right)}\, x^{4} a^{4}$$

$$-720 \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\, \sqrt{\left(a x-1\right) \left(a x+1\right)}}{\sqrt{a^{2}}}\right) x^{4} a^{5}-342 \sqrt{a^{2}}\, \left(\left(a x-1\right) \left(a x+1\right)\right)^{3/2} x^{2} a^{2}-1575 \sqrt{a^{2}}\, \sqrt{\left(a x-1\right) \left(a x+1\right)}\, x^{3} a^{3}$$

$$-720 \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\, \sqrt{\left(a x-1\right) \left(a x+1\right)}}{\sqrt{a^{2}}}\right) x^{3} a^{4}-3 \sqrt{a^{2}}\, \left(\left(a x-1\right) \left(a x+1\right)\right)^{3/2} x a+1575 a^{2} \sqrt{\left(a x-1\right) \left(a x+1\right)}\, \sqrt{a^{2}}\, x^{2}$$

$$+720 a^{3} \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\, \sqrt{\left(a x-1\right) \left(a x+1\right)}}{\sqrt{a^{2}}}\right) x^{2}+243 \left(\left(a x-1\right) \left(a x+1\right)\right)^{3/2} \sqrt{a^{2}}+525 \sqrt{a^{2}}\, \sqrt{\left(a x-1\right) \left(a x+1\right)}\, x a$$

$$+240 \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\, \sqrt{\left(a x-1\right) \left(a x+1\right)}}{\sqrt{a^{2}}}\right) x a^{2}-525 \sqrt{a^{2}}\, \sqrt{\left(a x-1\right) \left(a x+1\right)}-240 \ln\left(\frac{a^{2} x+\sqrt{a^{2}}\, \sqrt{\left(a x-1\right) \left(a x+1\right)}}{\sqrt{a^{2}}}\right) a\Bigg)$$

Problem 217: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(\frac{a x-1}{a x+1}\right)^{3/2}\left(c-\frac{c}{a^2 x^2}\right)^4}\, dx$$

Optimal(type 3, 279 leaves, 12 steps):

$$-\frac{10}{9 a c^4 \left(1-\frac{1}{a x}\right)^{9/2}\left(1+\frac{1}{a x}\right)^{3/2}}-\frac{29}{21 a c^4 \left(1-\frac{1}{a x}\right)^{7/2}\left(1+\frac{1}{a x}\right)^{3/2}}-\frac{208}{105 a c^4 \left(1-\frac{1}{a x}\right)^{5/2}\left(1+\frac{1}{a x}\right)^{3/2}}$$
$$-\frac{1147}{315 a c^4 \left(1-\frac{1}{a x}\right)^{3/2}\left(1+\frac{1}{a x}\right)^{3/2}}+\frac{x}{c^4 \left(1-\frac{1}{a x}\right)^{9/2}\left(1+\frac{1}{a x}\right)^{3/2}}+\frac{3 \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a x}}\,\sqrt{1+\frac{1}{a x}}\right)}{a c^4}$$
$$-\frac{1462}{105 a c^4 \left(1+\frac{1}{a x}\right)^{3/2}\sqrt{1-\frac{1}{a x}}}+\frac{2609\sqrt{1-\frac{1}{a x}}}{315 a c^4 \left(1+\frac{1}{a x}\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{a x}}}{315 a c^4 \sqrt{1+\frac{1}{a x}}}$$

Result(type 3, 765 leaves):

$$-\frac{1}{40320 a \sqrt{a^2}\,(a x-1)^4 c^4 \sqrt{(a x-1)(a x+1)}\,(a x+1)^4 \left(\frac{a x-1}{a x+1}\right)^{3/2}}\Bigg(-138915\sqrt{(a x-1)(a x+1)}\,\sqrt{a^2}\, x^9 a^9$$
$$-120960 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x^9 a^{10}+98595\left((a x-1)(a x+1)\right)^{3/2}\sqrt{a^2}\, x^7 a^7+416745\sqrt{(a x-1)(a x+1)}\,\sqrt{a^2}\, x^8 a^8$$
$$+362880 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x^8 a^9-75113\left((a x-1)(a x+1)\right)^{3/2}\sqrt{a^2}\, x^6 a^6-240861\left((a x-1)(a x+1)\right)^{3/2}\sqrt{a^2}\, x^5 a^5$$
$$-1111320\sqrt{(a x-1)(a x+1)}\,\sqrt{a^2}\, x^6 a^6-967680 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x^6 a^7+178863\left((a x-1)(a x+1)\right)^{3/2}\sqrt{a^2}\, x^4 a^4$$
$$+833490\sqrt{(a x-1)(a x+1)}\,\sqrt{a^2}\, x^5 a^5+725760 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x^5 a^6+252497\left((a x-1)(a x+1)\right)^{3/2}\sqrt{a^2}\, x^3 a^3$$
$$+833490\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}\, x^4 a^4+725760 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x^4 a^5-182307\sqrt{a^2}\left((a x-1)(a x+1)\right)^{3/2} x^2 a^2$$
$$-1111320\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}\, x^3 a^3-967680 \ln\left(\frac{a^2 x+\sqrt{a^2}\,\sqrt{(a x-1)(a x+1)}}{\sqrt{a^2}}\right) x^3 a^4-101271\sqrt{a^2}\left((a x-1)(a x+1)\right)^{3/2} x a$$

$$+ 74077 \left((a x - 1) (a x + 1) \right)^{3/2} \sqrt{a^2} + 416745 \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)}\, x a + 362880 \ln\left(\frac{a^2 x + \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)}}{\sqrt{a^2}} \right) x a^2$$

$$- 138915 \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)} - 120960 \ln\left(\frac{a^2 x + \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)}}{\sqrt{a^2}} \right) a \Bigg)$$

Problem 223: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(\frac{a x - 1}{a x + 1} \right)^{3/2}}{c - \frac{c}{a^2 x^2}}\, dx$$

Optimal(type 3, 124 leaves, 7 steps):

$$-\frac{3 \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a x}} \sqrt{1 + \frac{1}{a x}} \right)}{a c} + \frac{5 \sqrt{1 - \frac{1}{a x}}}{3 a c \left(1 + \frac{1}{a x} \right)^{3/2}} + \frac{x \sqrt{1 - \frac{1}{a x}}}{c \left(1 + \frac{1}{a x} \right)^{3/2}} + \frac{14 \sqrt{1 - \frac{1}{a x}}}{3 a c \sqrt{1 + \frac{1}{a x}}}$$

Result(type 3, 345 leaves):

$$-\frac{1}{3 a \sqrt{a^2} (a x + 1) c (a x - 1) \sqrt{(a x - 1) (a x + 1)}} \Bigg(\Bigg(-9 \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)}\, x^3 a^3 + 9 \ln\left(\frac{a^2 x + \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)}}{\sqrt{a^2}} \right) x^3 a^4$$

$$+ 6 \sqrt{a^2} \left((a x - 1) (a x + 1) \right)^{3/2} x a - 27 a^2 \sqrt{(a x - 1) (a x + 1)} \sqrt{a^2}\, x^2 + 27 a^3 \ln\left(\frac{a^2 x + \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)}}{\sqrt{a^2}} \right) x^2 + 5 \left((a x - 1) (a x \right.$$

$$\left. + 1) \right)^{3/2} \sqrt{a^2} - 27 \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)}\, x a + 27 \ln\left(\frac{a^2 x + \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)}}{\sqrt{a^2}} \right) x a^2 - 9 \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)}$$

$$+ 9 \ln\left(\frac{a^2 x + \sqrt{a^2} \sqrt{(a x - 1) (a x + 1)}}{\sqrt{a^2}} \right) a \Bigg) \left(\frac{a x - 1}{a x + 1} \right)^{3/2} \Bigg)$$

Problem 228: Result more than twice size of optimal antiderivative.

$$\int \frac{(a x + 1) \left(c - \frac{c}{a^2 x^2} \right)^{3/2}}{a x - 1}\, dx$$

Optimal(type 3, 185 leaves, 11 steps):

$$\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{-ax+1}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(-ax+1)(ax+1)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(ax+1)}{2(-ax+1)}+\frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\arcsin(ax)}{(-ax+1)^{3/2}(ax+1)^{3/2}}$$
$$+\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\operatorname{arctanh}\left(\sqrt{-ax+1}\sqrt{ax+1}\right)}{2(-ax+1)^{3/2}(ax+1)^{3/2}}$$

Result(type 3, 454 leaves):

$$\frac{1}{6\left(\frac{c(a^2x^2-1)}{a^2}\right)^{3/2}ca^2\sqrt{-\frac{c}{a^2}}}\left(\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{3/2}x\left(12a^5x^3\left(\frac{c(a^2x^2-1)}{a^2}\right)^{3/2}c\sqrt{-\frac{c}{a^2}}-12a^5\left(\frac{c(a^2x^2-1)}{a^2}\right)^{5/2}x\sqrt{-\frac{c}{a^2}}\right.\right.$$
$$+4a^4\left(\frac{(ax-1)c(ax+1)}{a^2}\right)^{3/2}cx^2\sqrt{-\frac{c}{a^2}}-a^4\left(\frac{c(a^2x^2-1)}{a^2}\right)^{3/2}cx^2\sqrt{-\frac{c}{a^2}}+6a^3c^2\sqrt{\frac{(ax-1)c(ax+1)}{a^2}}x^3\sqrt{-\frac{c}{a^2}}$$
$$-3a^4\left(\frac{c(a^2x^2-1)}{a^2}\right)^{5/2}\sqrt{-\frac{c}{a^2}}-18a^3c^2x^3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\sqrt{-\frac{c}{a^2}}+18c^{5/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)x^2a\sqrt{-\frac{c}{a^2}}$$
$$-6c^{5/2}\ln\left(\frac{\sqrt{\frac{(ax-1)c(ax+1)}{a^2}}\sqrt{c}+cx}{\sqrt{c}}\right)x^2a\sqrt{-\frac{c}{a^2}}+3c^2\sqrt{\frac{c(a^2x^2-1)}{a^2}}x^2a^2\sqrt{-\frac{c}{a^2}}$$
$$\left.\left.+3c^3\ln\left(\frac{2\left(\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c(a^2x^2-1)}{a^2}}a^2-c\right)}{xa^2}\right)x^2\right)\right)$$

Problem 229: Result more than twice size of optimal antiderivative.

$$\int\frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$$

Optimal(type 3, 109 leaves, 7 steps):

$$-\frac{(ax+1)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(-2ax+5)(-ax+1)(ax+1)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}-\frac{2(-ax+1)^{3/2}(ax+1)^{3/2}\arcsin(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

Result(type 3, 325 leaves):

$$\frac{1}{3\sqrt{\frac{(ax-1)c(ax+1)}{a^2}}\,x^3\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{3/2}a^4c^{3/2}}\Bigg(\Bigg(3\sqrt{\frac{(ax-1)c(ax+1)}{a^2}}\,c^{3/2}x^3a^3-15x^2a^2\sqrt{\frac{(ax-1)c(ax+1)}{a^2}}\,c^{3/2}+4\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,c^{3/2}x^2a^2+6\sqrt{\frac{(ax-1)c(ax+1)}{a^2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,xa^2c-4\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,c^{3/2}xa-6\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)c\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a\sqrt{\frac{(ax-1)c(ax+1)}{a^2}}+12\sqrt{\frac{(ax-1)c(ax+1)}{a^2}}\,c^{3/2}-2\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,c^{3/2}\Bigg)(ax+1)\Bigg)$$

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Problem 237: Result more than twice size of optimal antiderivative.
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$$\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^2}\,dx$$

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Optimal(type 3, 91 leaves, 7 steps):
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$$\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{2}+\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2x}+\frac{3a^2x\,\mathrm{arctanh}\left(\sqrt{-ax+1}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{-ax+1}\sqrt{ax+1}}$$

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Result(type 3, 347 leaves):
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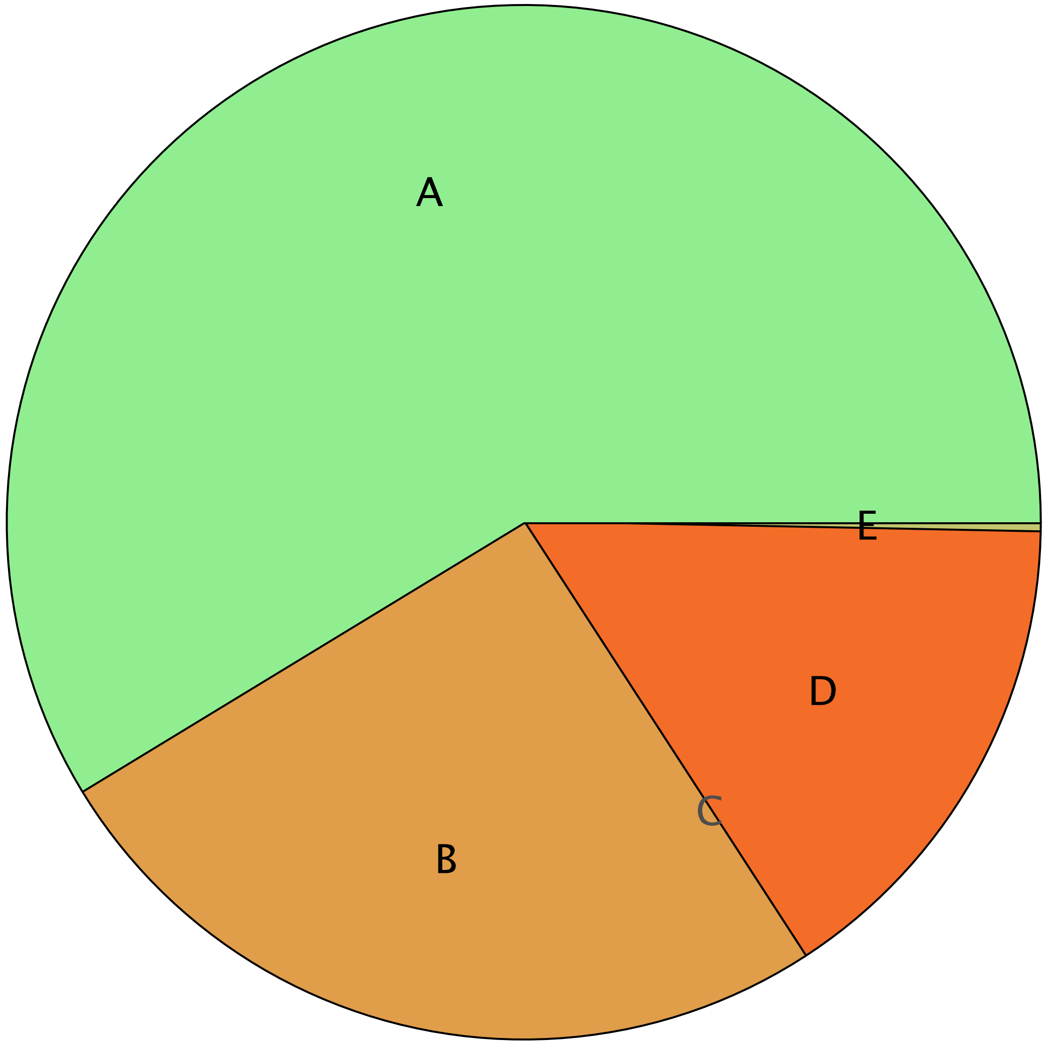
$$\frac{1}{2x\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,c\sqrt{-\frac{c}{a^2}}}\Bigg(\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\Bigg(4a^3x^3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,c\sqrt{-\frac{c}{a^2}}-4a^3\left(\frac{c(a^2x^2-1)}{a^2}\right)^{3/2}x\sqrt{-\frac{c}{a^2}}+4a^2\sqrt{\frac{(ax-1)c(ax+1)}{a^2}}\,cx^2\sqrt{-\frac{c}{a^2}}-4ac^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)x^2\sqrt{-\frac{c}{a^2}}+4ac^{3/2}\ln\left(\frac{\sqrt{\frac{(ax-1)c(ax+1)}{a^2}}\sqrt{c}+cx}{\sqrt{c}}\right)x^2\sqrt{-\frac{c}{a^2}}-3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2cx^2\sqrt{-\frac{c}{a^2}}-a^2\left(\frac{c(a^2x^2-1)}{a^2}\right)^{3/2}\sqrt{-\frac{c}{a^2}}-3c^2\ln\left(\frac{2\left(\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2-c\right)}{xa^2}\right)x^2\Bigg)\Bigg)$$

Summary of Integration Test Results

322 integration problems



A - 189 optimal antiderivatives
B - 82 more than twice size of optimal antiderivatives
C - 0 unnecessarily complex antiderivatives
D - 50 unable to integrate problems
E - 1 integration timeouts